Math 265, Practice Midterm 2

Name: ______________________________

Section: __________

This exam consists of 8 pages including this front page.

Ground Rules

1. No calculator is allowed.
2. Show your work for every problem unless otherwise stated.
3. You may use one 4-by-6 index card, both sides.

Score		
1	15	
2	20	
3	15	
4	20	
5	15	
6	15	
Total	100	

1. The following are true/false questions. You don't have to justify your answers. Just write down either T or F in the table below. A, B, C, X, b are always matrices here.

(a) It is not possible to find 4 linearly independent vectors v_1, v_2, v_3, v_4 in $\mathbb{R}^3$.

(b) Let A be an $m \times n$ matrix. If the dimension of the column space of A is 5, then $n \geq 5$.

(c) Consider a system of linear equations $AX = b$ where A is a square matrix. If $\det(A) = 0$ then the system is inconsistent.

(d) Let V be an inner product space and $u, v, w \in V$. If w is orthogonal to u and v, then w is orthogonal to any linear combination of u and v.

(e) Let A be an $n \times n$-matrix. If $\text{rank}(A) < n$ then $\det(A) = 0$.

	(a)	(b)	(c)	(d)	(e)
Answer					

2. Quick Questions, $A,\ B,\ C,\ X,\ b$ are always matrices here:

(a) Suppose $u = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Find the length of u and v.

(b) Let u and v be as the above question and θ is the angle between u and v. Find $\cos(\theta)$.

(c) Find a basis for the space of 3×3 real symmetric matrices.

(d) Let A be a 3×5-matrix. What are the possible values of nullity of A?

3. (a) For what values of c are the vectors $\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ c \end{pmatrix}$ in $\mathbb{R}^3$ linearly independent?

(b) If possible, find a, b, c so that $u = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is orthogonal to $v = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

4. Let $A = \begin{pmatrix} 1 & 0 & -1 & 1 \\ 1 & -1 & 0 & -1 \\ 2 & -1 & -1 & 0 \end{pmatrix}$.

1. Find a basis for row space of A.

2. Find a basis for the column space of A.

3. Find a basis for the null space of A.

4. Verify the equality $\text{rank}(A) + \text{Nullity}(A) = n$.

5. Explain why the columns of A are linearly dependent.

5. Let $V \subset \mathbb{R}^4$ be a subspace spanned by $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$.

1. Find a orthonormal basis of V.

2. Let $u = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$. Find $\mathrm{Proj}_V u$.

6. Consider the following system of linear equation

$$\begin{aligned} -2x_1 + 3x_2 - x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 4 \\ -2x_1 - x_2 + x_3 &= -3 \end{aligned}$$

1. Find the rank of the coefficient matrix.

2. Find the rank of the augmented matrix.

3. Is the system consistent? why?

7. Let A be a 3×5-matrix.

1. What will be the maximal possible rank of A.

2. Show that columns of A are linearly independent.

3. what will be the maximal possible dimension of $N(A)$.