Math 265 Midterm 2

Name: ______________________________

Ground Rules

1. No calculator is allowed.
2. Show your work for every problem unless otherwise stated.
3. You may use one 4-by-6 index card, both sides.

Score		
1	15	
2	20	
3	15	
4	20	
5	15	
6	15	
Total	100	

1. The following are true/false questions. You don't have to justify your answers. Just write down either T or F in the table below. A, B, C, X, b are always matrices here.

(a) Any 5 vectors v_1, v_2, v_3, v_4, v_5 in $\mathbb{R}^4$ are linearly dependent.

(b) Let A be an $m \times n$ matrix. If the rows of A are linear independent, so are the columns.

(c) Consider a system of linear equations $AX = b$ where A is a $n \times n$-matrix. If the system is inconsistent then $\text{rank}(A) < n$.

(d) Let V be an inner product space, $W \subset V$ a subspace and $w \in W$, $v \in V$ nonzero vectors. Then $v \in W^\perp$ if and only if $v \perp w$.

(e) Let A be an $n \times n$-matrix. If $\det(A) = 0$ then the dimension of the column space of A is less than n .

	(a)	(b)	(c)	(d)	(e)
Answer					

2. Quick Questions, $A,\ B,\ C,\ X,\ b$ are always matrices here:

(a) Suppose $u = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$. Let θ be the angle between u and v. Find $\cos(\theta)$.

(b) Assume that u, v are vectors in a inner product space with the inner product $\langle , \rangle$. Suppose that $\| u \| = 1$, $\| v \| = 2$ with $u \perp v$. Compute $\| u + v \| = \sqrt{\langle u + v, u + v \rangle} = ?$

(c) Find a basis for the space of 3×3 real skew-symmetric matrices (recall that A is skew-symmetric if $A^T = -A$).

(d) Let A be a 3×5-matrix. What are the possible values of the nullity of A?

3. (a) For what values of c are the vectors $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \\ c \end{pmatrix}$ in $\mathbb{R}^3$ linearly independent?

(b) Suppose that W is the subspace of $\mathbb{R}^3$ which is spanned by $u = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$. Find a basis of $W^{\perp}$.

4. Let $A = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ 2 & 2 & -1 & 0 \end{pmatrix}$.

1. Find a basis for the row space of A.

2. Find a basis for the column space of A.

3. Find a basis for the null space of A.

4. Verify the equality $\text{rank}(A) + \text{Nullity}(A) = n$.

5. Explain why the rows of A are linearly dependent.

5. Let $V \subset \mathbb{R}^4$ be a subspace spanned by $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \right\}$.

1. Find a orthonormal basis of V.

2. Let $u = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$. Find $\mathrm{Proj}_V u$.

6. Consider the following system of linear equation

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + 2x_2 &= 2 \\ x_1 - x_2 &= 0 \end{aligned}$$

1. Find the ranks of the coefficient matrix and the augmented matrix respectively.

2. Is the system consistent? Why?

3. If the system is inconsistent. Compute the least squares solution.