QUIZ 4

A mass weighting 32 lb stretches a spring 16 feet. The mass is attached to viscous damper with a damping constant 2 lb-sec/ft. The mass is pulled down an additional 3 in, and then released. Let $u = u(t)$ denote the displacement of the mass from the equilibrium. (The gravity constant is $g = 32ft/sec^2$.)

(1) Find the initial value problem that $u(t)$ satisfies.
Solution: We have $mg = 32$ and $L = 16$. Then $m = 32/32 = 1$ and $k = mg/L = 2$. Note that $u(0) = 3$ in $= 0.25$ foot, and $\gamma = 2$. We have the initial value problem as the following

$$u''(t) + 2u'(t) + 2u(t) = 0, \quad u(0) = 0.25, \ u'(0) = 0.$$

(2) Find $u(t)$.
Solutions: The characteristic polynomial $f(r) = r^2 + 2r + 2 = (r+1)^2 + 1$. Hence $f(r)$ has root $r = -1 \pm i$. Then

$$u(t) = c_1 e^{-t} \cos t + c_2 e^{-t} \sin t.$$

Now $u'(t) = -c_1 e^{-t} \cos t - c_1 e^{-t} \sin t - c_2 e^{-t} \sin t + c_2 e^{-t} \cos t$. Plug in the initial value $u(0) = 0.25$ and $u'(0) = 0$. We have $c_1 = 0.25$ and $c_2 = 0.25$. So

$$u(t) = 0.25 e^{-t} \cos t + 0.25 e^{-t} \sin t.$$

(3) What is the long run behavior of $u(t)$? Namely, what is

$$\lim_{t \to +\infty} u(t)?$$

Solutions: Obviously $\lim_{t \to +\infty} u(t) = 0$ from the above answer.