MA 584 HW 8 DUE DEC. 11TH

(1) Let K be the real quadratic field $\mathbb{Q}(\sqrt{d})$ with $d > 0$ a square-free integer. We may find a fundamental unit ϵ in the following steps:
  (a) Consider the Pell's equation $x^2 - dy^2 = \pm 4$.
  (b) Check one by one $y = 1, 2, \ldots$ to find the smallest solution $x_0 > 0, y_0 > 0$ of the above equations. Check the equation $x^2 - dy^2 = -4$ before $x^2 - dy^2 = 4$ each time.
  (c) Then $\epsilon = \frac{x_0 + y_0\sqrt{d}}{2}$ is a fundamental unit of U_K.

Prove the above algorithm makes sense.

(2) Find a fundamental unit of $\mathbb{Q}(\sqrt{D})$ for $D = 5, 6, 7, 10$.

(3) Let K be a number field and $\mathbb{A}_K$ the ring of adéle. Show that $\mathbb{A}_K$ is locally compact.

(4) Let $\mathbb{I}_K$ be the group of idéle. Does the topology of $\mathbb{I}_K$ come from $\mathbb{A}_K$ as subspace? Why or why not?