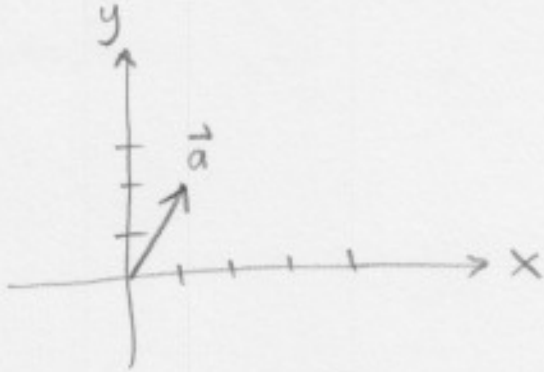
Section 1.1

Vectors in 2-dimensions

$\vec{a} = (a_1, a_2)$

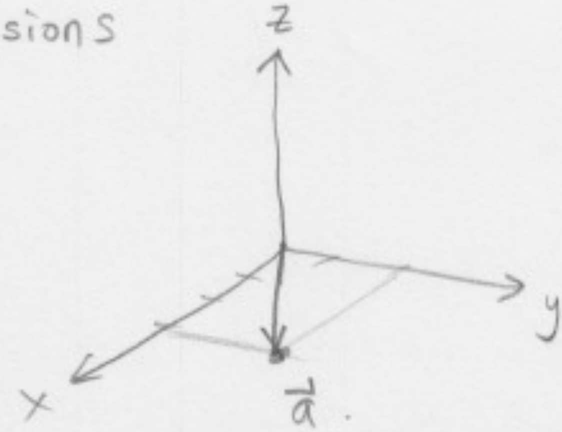
Ex $\vec{a} = (1, 2)$



Vectors in 3-dimensions

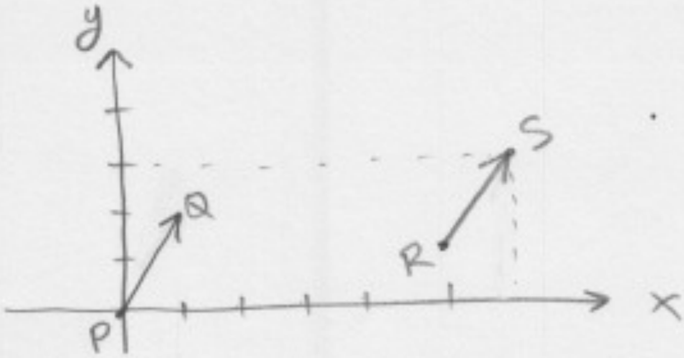
$\vec{a} = (a_1, a_2, a_3)$

Ex $\vec{a} = (3, 2, 0)$



Two vectors are equal if they have the same magnitud and direction.

<u>Ex</u>:



$P = (0,0)$ $Q = (1,2)$ Vector $\overrightarrow{PQ} = (1,2)$

$R = (5,1)$ $S = (6,3)$ Vector $\overrightarrow{RS} = (1,2)$

P, Q, R, S are <u>points</u>, not vectors. A vector has a magnitud and direction. We have:

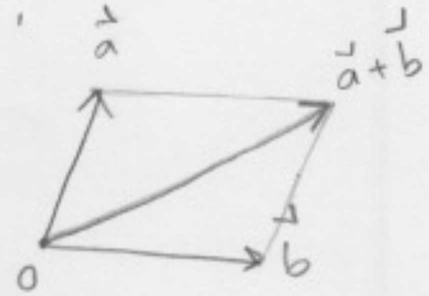
$$\overrightarrow{PQ} = \overrightarrow{RS}$$

Operations with vectors

* <u>Addition</u> :

$\vec{a} = (1,0)$, $\vec{b} = (2,3)$

$\vec{a}+\vec{b} = (3,3)$



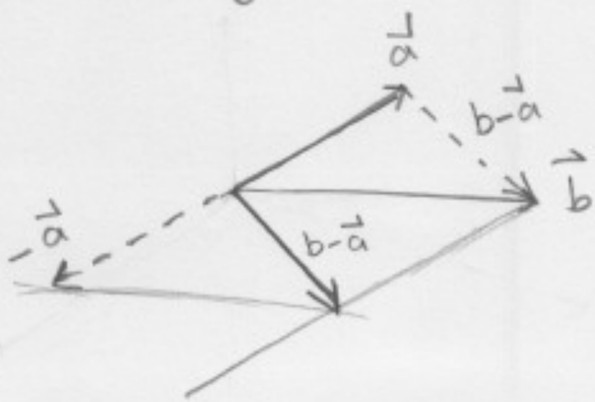
Geometric meaning of addition.

* <u>Subtraction</u> :

$\vec{b} - \vec{a} = (2-1, 3-0) = (1,3)$

Geometrically

$\vec{b} - \vec{a} = \vec{b} + (-\vec{a})$



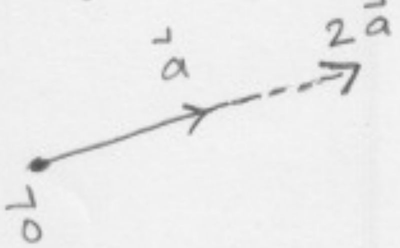
* Multiplication by a scalar number.

$\vec{a} = (1, 0)$

$5\vec{a} = 5(1,0) = (5,0)$

Geometrically:

The vector "magnitud" is 5 times largers.



The standard basis vectors:

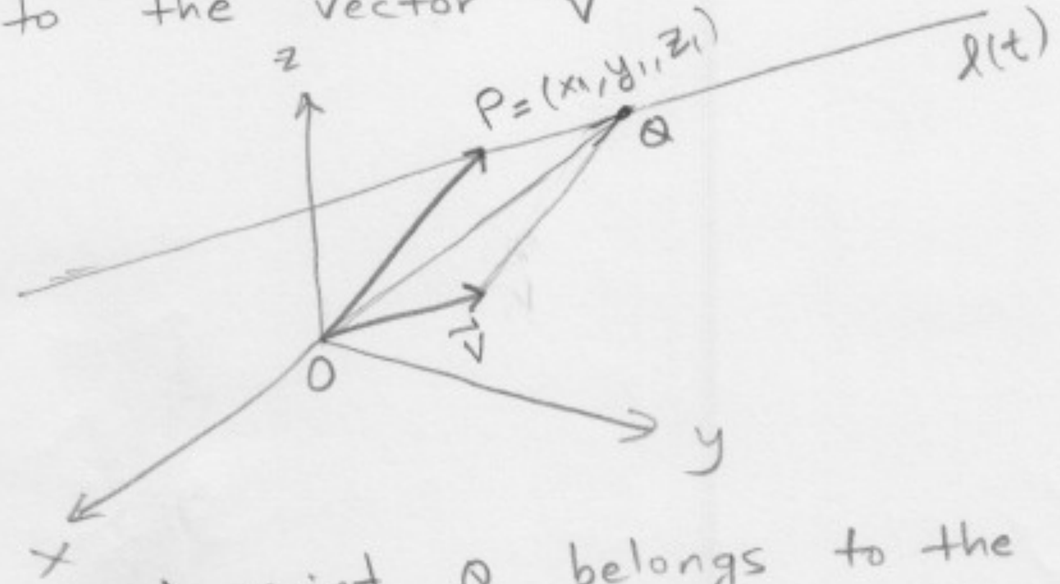
$\vec{i} = (1,0,0)$ $\vec{j} = (0,1,0)$, $\vec{k} = (0,0,1)$

If $\vec{a} = (a_1, a_2, a_3)$, where a_1, a_2, a_3 are the components of the vectors, then we can also write:

$$\begin{aligned}\vec{a} &= a_1\vec{i} + a_2\vec{j} + a_3\vec{k} \\ &= a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1) \\ &= (a_1, a_2, a_3).\end{aligned}$$

Equations of Lines:

Consider the vector $\overrightarrow{OP} = (x_1, y_1, z_1)$, where $O = (0,0,0)$ and $P = (x_1, y_1, z_1)$, and let $\vec{v} = (v_1, v_2, v_3)$. We want to obtain the equation of the line that contains the point P and is parallel to the vector $\vec{v}$



A point Q belongs to the the line $\ell(t)$ if and only if Q is such that:

$$\overrightarrow{OQ} = \overrightarrow{OP} + t\vec{v} = (x_1, y_1, z_1) + t(v_1, v_2, v_3)$$

for some number t.

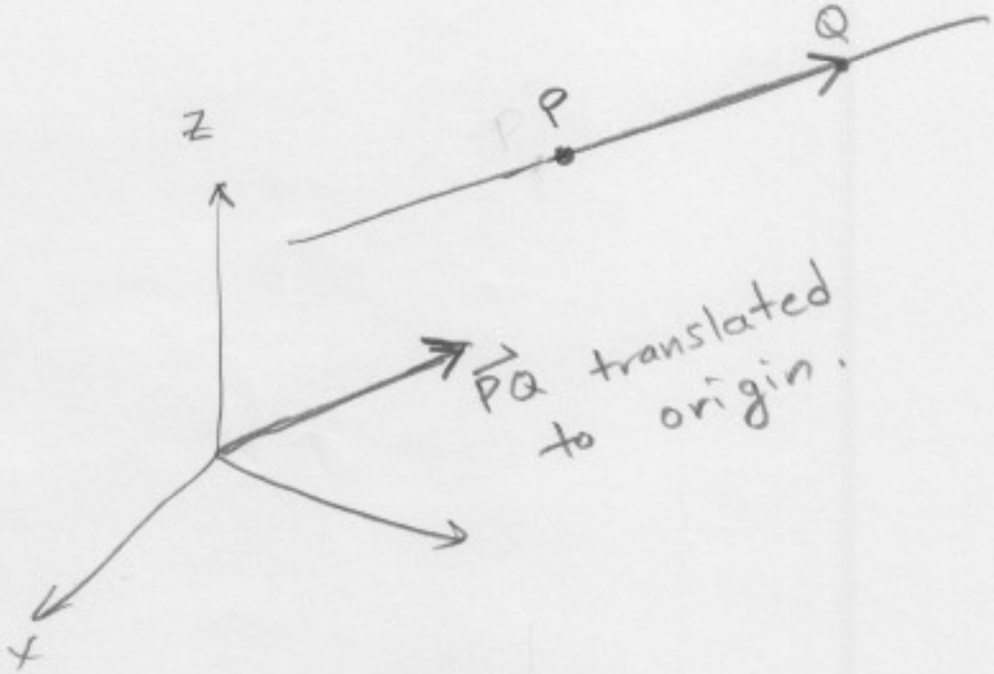
Hence, the equation of $\ell(t)$ is:

$$\ell(t) = (x(t), y(z), z(t))$$

$$x(t) = x_1 + tv_1$$
$$y(t) = y_1 + tv_2$$
$$z(t) = z_1 + tv_3$$

The parametric equation of the line $\ell(t)$ through $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ is:

$$x = x_1 + (x_2 - x_1)t$$
$$y = y_1 + (y_2 - y_1)t$$
$$z = z_1 + (z_2 - z_1)t$$



<u>Ex</u>: Find the equation of the line through the point $(3,-1,2)$ in the direction $2i-3j+4k$

$$x = 3+2t$$
$$y = -1-3t$$
$$z = 2+4t$$

Planes

Let $\vec{v}$ and $\vec{w}$ any two vectors. Then the plane formed by these two vectors is the set of all points that are the "tips" of vectors of the form

$s\vec{v} + t\vec{w}$, where s, t are any real numbers.