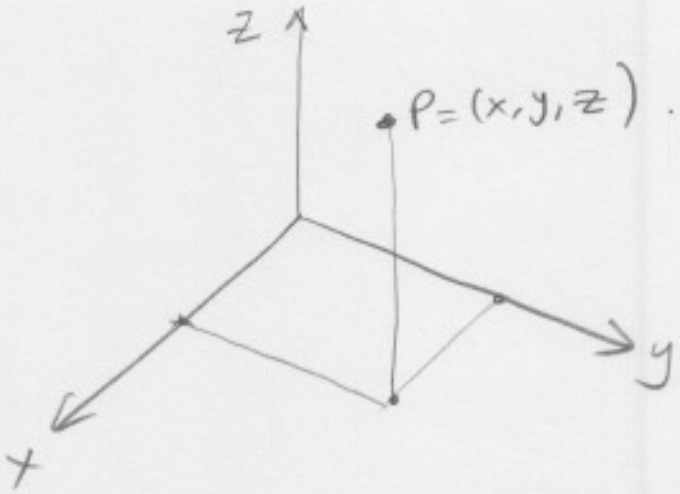
Section 1.4.

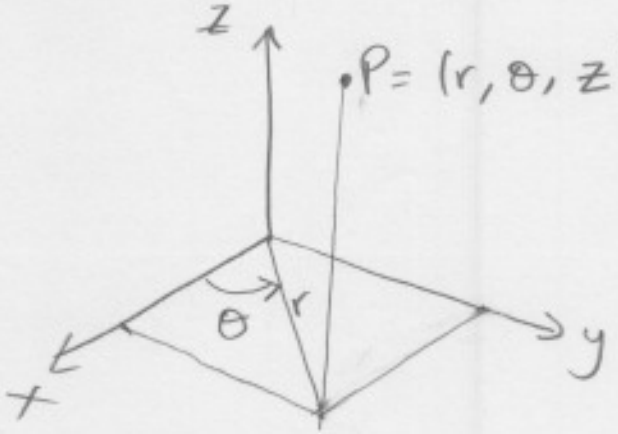
Cylindrical, Euclidean and spherical coordinates.

- Euclidean Coordinates in $\mathbb{R}^3$

$P = (x, y, z)$



- Cylindrical coordinates in $\mathbb{R}^3$



$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$0 \leq \theta < 2\pi$

$r \geq 0$.

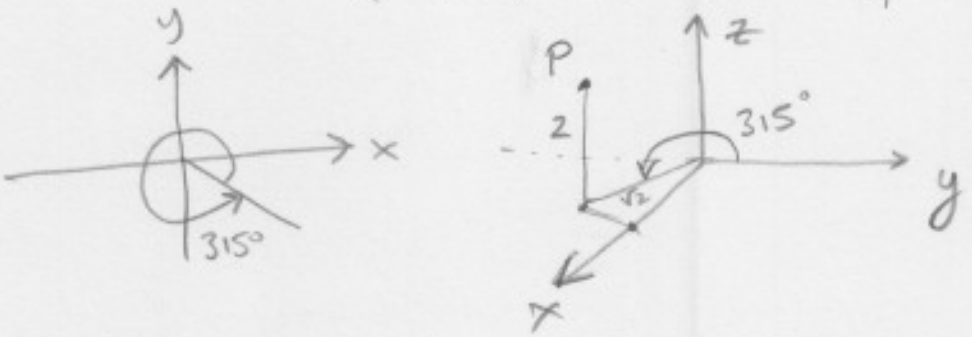
Ex: The point $P=(1,-1,2)$ is in Euclidean coordinates. Find coordinates in cylindrical coordinates.

$$r = \sqrt{x^2+y^2} = \sqrt{2}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$
$$1 = \sqrt{2}\cos\theta \qquad -1 = \sqrt{2}\sin\theta$$
$$\cos\theta = \frac{1}{\sqrt{2}} \qquad \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\theta = \frac{7\pi}{4}\left(\frac{180}{\pi}\right) = 315$$

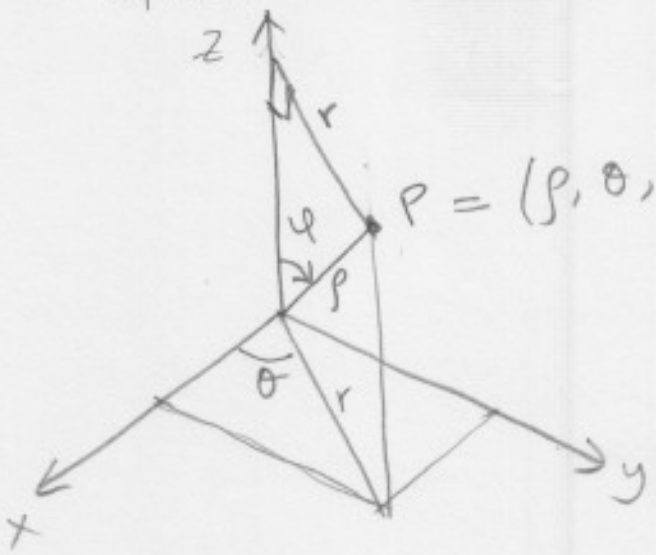
$$P = (r,\theta,z) = \left(\sqrt{2}, \frac{7\pi}{4}, 2\right).$$



Ex: The point $\left(2, \frac{\pi}{6}, 3\right)$ in cylindrical coordinates correspond to $(\sqrt{3}, 1, 3)$ in Euclidean coordinates, since:

$$r = 2 \qquad x = r\cos\theta = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$$
$$\theta = \frac{\pi}{6} \qquad y = r\sin\theta = 2\cdot\frac{1}{2} = 1.$$

Spherical coordinates



$P = (\rho, \theta, \varphi)$

$0 \le \theta < 2\pi$

$0 \le \varphi < \pi$

$\rho \ge 0$

θ, φ not defined when $\rho = 0$.

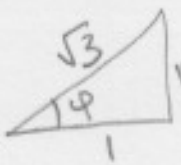
$$\begin{cases} x = r\cos\theta = \rho \sin\varphi \cos\theta \\ y = r\sin\theta = \rho \sin\varphi \sin\theta \\ z = \rho\cos\varphi \end{cases}$$

$$x^2+y^2+z^2 = \rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta + \rho^2\cos^2\varphi$$
$$= \rho^2\sin^2\varphi \cdot (1) + \rho^2\cos^2\varphi$$
$$= \rho^2$$

Ex: $P = (1, -1, 1)$ is in Eucledian coordinates. Write P in spherical coordinates.

$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{3}$

$\cos\varphi = \frac{1}{\sqrt{3}}$ $\quad \varphi = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$



$\sin\varphi = \frac{\sqrt{2}}{\sqrt{3}}$

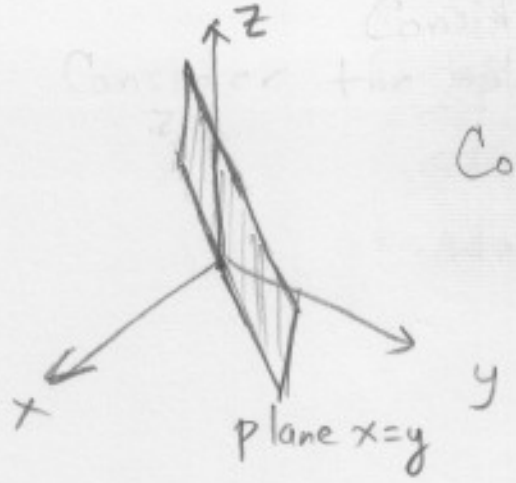
$\cos\theta = \frac{1}{\sqrt{3}\sin\varphi} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{-1}{\sqrt{3}\sin\varphi} = \frac{-1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{7\pi}{4}$

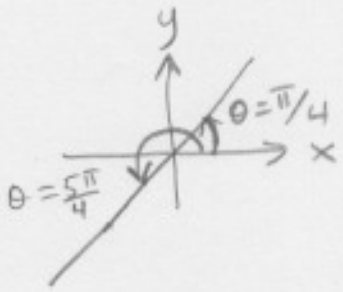
$\therefore$ $\boxed{P = \left(\sqrt{3}, \frac{7\pi}{4}, \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)}$

Ex:



Consider the plane $x=y$

Equation of plane $x=y$ in cylindrical coordinates is $\theta = \frac{\pi}{4}$ and $\theta = \frac{5\pi}{4}$



Equation of plane $x=y$ in spherical coordinates is: $\theta = \frac{\pi}{4}$ and $\theta = \frac{5\pi}{4}$.

Ex: Eq. of sphere $x^2+y^2+z^2=4$ in spherical coordinates is $\rho = 2$.