Math 261, Lecture 1, 8/20/18

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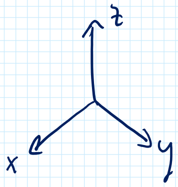
W 5:30 - 6:30 or by appt
F 11:30 - 12:30

Outline §§ 12.1 - 12.4 Review

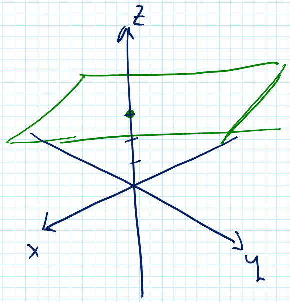
§ 12.1 3D Coordinates



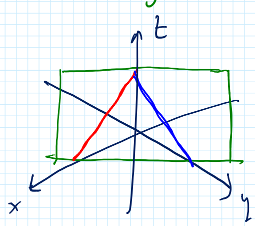
"right hand rule"

$$\boxed{\begin{aligned}&\text{Distance } (x_1,y_1,z_1) \text{ to } (x_2,y_2,z_2)\\ &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}\end{aligned}}$$

Surfaces Ex. plane $z=3$



Ex. plane $x+y+z=4$



$y=0 \quad z=4-x$

$x=0 \quad z=4-y$

Equation of a Sphere

distance² to (x_0,y_0,z_0)

standard form $\boxed{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2}$

(x_0,y_0,z_0) = Center

r = Radius

Problem. Find the sphere given by the equation

$$x^2+y^2+z^2 = 4x-2y+6z+2$$

Solution. collect like terms move x,y,z's to LHS

$$x^2-4x+y^2+2y+z^2-6z=2$$

$$(x^2-4x+\boxed{4})+(y^2+2y+1)+(z^2-6z+9)=2$$

complete the square $-2a=-4$, $a=2$, $a^2=4$ $\quad 2a=2$, $a=1$, $a^2=1$ $\quad -2a=-6$, $a=3$, $a^2=9$

$$(x-2)^2+(y+1)^2+(z-3)^2=2+4+1+9=16$$

$x^2-2ax+a^2$
$=(x-a)^2$

$x^2+2ax+a^2$
$=(x+a)^2$

$x^2 + 2ax + a^2$
$= (x+a)^2$

complete the square $\quad -2a = -4 \quad\quad 2a = 2 \quad\quad -2a = -6$

$a = 2,\ a^2 = 4 \quad\quad a = 1,\ a^2 = 1 \quad\quad a = 3,\ a^2 = 9$

$$(x-2)^2 + (y+1)^2 + (z-3)^2 = 2 + \underbrace{4+1+9}_{\text{add same to RHS}} = 16$$

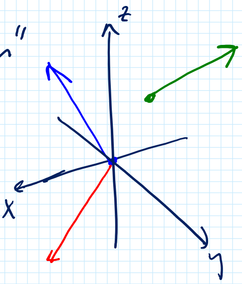
Center $= (x_0, y_0, z_0) = (2,-1,3)$

Radius $= r = 4$

Ex. Intersect with $z = 4$

§ 12.2 Vectors

"vector = magnitude + direction"



vectors at origin

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$\boxed{\begin{array}{l} * \text{ zero vector} \\ \vec{0} = \langle 0,0,0 \rangle \end{array}}$

Addition $\quad \vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

Scalar Mult. $\quad c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$

Ex. $\vec{a} = \langle 2,-1,0 \rangle$, $\vec{b} = \langle -1,3,5 \rangle$, $c = 4$

$$\vec{a} - 2\vec{b} = \langle 2,-1,0 \rangle - 2\langle -1,3,5 \rangle = \langle 2,-1,0 \rangle + \langle 2,-6,-10 \rangle$$
$$= \langle 4,-7,-10 \rangle$$

Magnitude (length) $\boxed{|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}}$

Ex. $\vec{a} = \langle 2,2,-3 \rangle$

$$|\vec{a}| = \sqrt{2^2 + 2^2 + (-3)^2} = \sqrt{4+4+9} = \sqrt{17}$$

Standard basis vectors

$$i = \langle 1,0,0 \rangle,\ j = \langle 0,1,0 \rangle,\ k = \langle 0,0,1 \rangle$$

$$\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1 i + a_2 j + a_3 k$$

i, j, k are special!

§ 12.3 Dot Products

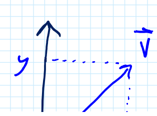
$* \text{ 2D} \quad \langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$

$\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$

Dot Product $\boxed{\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3}$

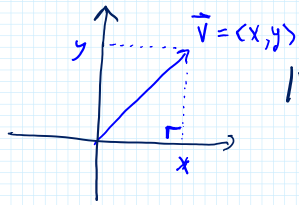
Ex.

2D case



$$|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{\langle x,y \rangle \cdot \langle x,y \rangle} = \sqrt{x^2 + y^2}$$

Fact. $\boxed{|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}}$



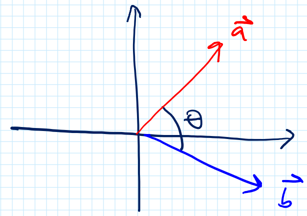
2D case

$|\vec{v}| = \sqrt{\vec{v}\cdot\vec{v}} = \sqrt{\langle x,y\rangle\cdot\langle x,y\rangle} = \sqrt{x^2+y^2}$

Pythagorean theorem

Angle Formula

$$\boxed{\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}}$$



$\vec{a}, \vec{b} \neq \vec{0}$, orthogonal if $\vec{a}\cdot\vec{b} = 0$ $(\vec{a} \perp \vec{b})$

Ex.

Ex.

§ 12.4 Cross Product

$\vec{a} = \langle a_1, a_2, a_3\rangle \quad \vec{b} = \langle b_1, b_2, b_3\rangle$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

determinant 3x3 matrix

* Determinant 2x2 matrix

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Ex. $\vec{a} = \langle 1, -2, -1\rangle$, $\vec{b} = \langle 3, 0, -2\rangle$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 3 & 0 & -2 \end{vmatrix} = Ai + Bj + Ck$$

$$A = \begin{vmatrix} \cancel{i} & \cancel{j} & \cancel{k} \\ \cancel{1} & -2 & -1 \\ \cancel{3} & 0 & -2 \end{vmatrix} = \begin{vmatrix} -2 & -1 \\ 0 & -2 \end{vmatrix} = (-2)(-2) - (-1)0$$
$$= 4$$

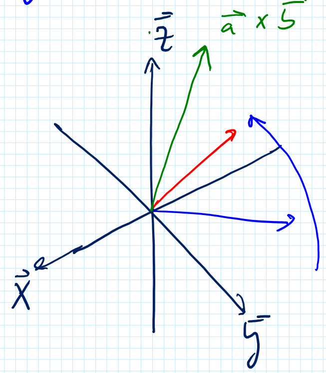
$$B = -\begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 3 & 0 & -2 \end{vmatrix} = -\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = (1(-2) - (-1)3) = -1$$

minus for j term

$$C = \begin{vmatrix} i & j & k \\ 1 & -2 & -1 \\ 3 & 0 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 3 & 0 \end{vmatrix} = 1\cdot 0 - (-2)3 = 6$$

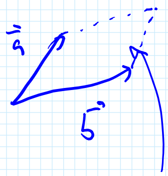
Answer $\vec{a} \times \vec{b} = 4i - j + 6k$

Fact. $\vec{a} \times \vec{b} \perp \vec{a}, \vec{b}$



By right hand rule blue vector is $\vec{a}$, red is $\vec{b}$

$\vec{a}$? $\vec{b}$?



Fact. $|\vec{a} \times \vec{b}|$ = Area of Parallelogram with sides $\vec{a}, \vec{b}$

Fact. $\boxed{\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}}$

Ex. Standard Coordinates

$i \times j = k$
$k \times j = -i$