Math 261, Lecture 18, 10/1/18

Announcements

- EXAM 1 is tomorrow, Oct. 2, 8pm, Elliott Hall
- Sit in the section assigned to your TA
- Please arrive 10 minutes early, at least.

Office Hours: Today, 3:30-5:30 MATH 744
Tues, 1:30-3:30

Today: Review, Lessons 2-16

Chapter 12.

- Lines point $p=(x_0,y_0,z_0)$, direction $\vec{v}=\langle a,b,c\rangle$

$$L = \begin{cases} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{cases}$$

- Planes point $p(x_0,y_0,z_0)$, $\underline{\text{normal}}$ $\vec{n}=\langle a,b,c\rangle$

direction $\perp$ to plane

P is given by the equation

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

* Normal is the x, y, z coefficients always

↳ direction of line of intersection is $\vec{n}_1 \times \vec{n}_2$

↳ angle of planes is $\dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \cos(\theta)$

517 #1 $P_1 \quad x - y + z = 1$

$P_2 \quad 2x + y + z = 4$

Find a line parallel to planes thru $(1,3,-3)$

Where does this line intersect $z = 0$?

$\vec{n}_1 = \langle 1,-1,1 \rangle \;,\; \vec{n}_2 = \langle 2,1,1 \rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} i - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} j + \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} k$$

$$= -2i + j + 3k$$

or $\langle -2, 1, 3 \rangle$

$$L = \begin{cases} x = 1 - 2t \\ y = 3 + t \\ z = -3 + 3t \end{cases} \xrightarrow{z=0} \begin{array}{l} 0 = -3 + 3t \\ t = 1 \end{array}$$

Answer, $x = 1 - 2 \cdot 1 = -1$, $y = 3 + 1 = 4$

So $(-1, 4, 0)$ or E

Quadric Surfaces "2+1+3"

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paraboloid

$$\underbrace{x^2 + 3y^2}_{\text{same sign = elliptic}} - z = 0$$

One variable is missing a square term

$$\underbrace{x^2 - 2x - 4y^2}_{\text{different signs = hyperbolic}} + z = 0$$

ellipse

$$x^2 + 5y^2 + z^2 > 0$$

all positive signs

All variables have square terms

$$x^2 - 5y^2 + z^2 = 0$$ cone

$$x^2 - 5y^2 + z^2 < 0$$ two sheeted hyperboloid

no solution $y=0$

$$x^2 - 5y^2 + z^2 > 0$$ one sheeted hyperboloid

ellipse is a solution

Works if only one minus term. If two, multiply both sides by -1.

S18, #1 Identify the surface defined by

$$x^2 - y^2 - 4x + z^2 = 4$$

$$x^2 - 4x \quad - y^2 \quad + z^2 = 4$$

Complete square

$x^2 - 4x + 4 \quad - y^2 \quad + z^2 = 4 + 4 \quad = 8$

$(x-2)^2 \quad - y^2 \quad + z^2 \quad = \quad 8$

* all terms quadratic
* one minus sign
* $= 8 > 0$

Answer one sheeted hyperboloid A

Chapter 13 $\quad \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

- $\vec{r}\,'(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle$ tangent vector

- unit tangent $\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

- unit normal $\vec{N}(t) = \dfrac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|}$

- curvature $\kappa(t) = \dfrac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$

- Velocity, speed, and acceleration
  ↳ Initial Value Problems

↳ Initial Value Problems

- Arc Length $\int_a^b |\vec{r}\,'(t)|\,dt$

F16, #1 $\vec{r}(t) = \langle 9\cos(t), 9\sin(t), 0 \rangle$

Find $K(\pi)$

$\vec{r}\,'(t) = \langle -9\sin t, 9\cos t, 0 \rangle$

$\vec{r}\,''(t) = \langle -9\cos t, -9\sin t, 0 \rangle$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} i & j & k \\ -9\sin & 9\cos & 0 \\ -9\cos & -9\sin & 0 \end{vmatrix}$$

$$= \begin{vmatrix} -9\sin & 9\cos \\ -9\cos & -9\sin \end{vmatrix} k$$

$$= (81\sin^2 + 81\cos^2)k = 81k$$

so $|\vec{r}\,'(t) \times \vec{r}\,''(t)| = 81$

$$|\vec{r}\,'(t)| = \sqrt{(-9\sin)^2 + (9\cos)^2 + 0^2}$$

$$= \sqrt{81\sin^2 + 81\cos^2} = \sqrt{81} = 9$$

Answer, $K(\pi) = \dfrac{81}{9^3} = \dfrac{1}{9}$ $\boxed{C}$

Chapter 14.

- Limits
  - ↳ To show exist, do algebra, then plug in to find a number
  - ↳ To show DNE try setting $x=0$ or $y=0$, then $y=mx$ if necessary

- Tangent plane to $z = f(x,y)$ at $z_0 = f(x_0, y_0)$ .3

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

- $\vec{\nabla} f = \langle f_x, f_y \rangle$

- Directional Derivative $D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$

↳ Need $|\vec{u}| = 1$

- Chain Rule

- 2^{nd} Derivative Test

- Tangent to Implicit Surface $F(x,y,z) = c$

If $F(x_0, y_0, z_0) = C$ then

tangent plane at (x_0, y_0, z_0) is

$$F_x(x_0,y_0,z_0)(x - x_0) + F_y(x_0,y_0,z_0)(y - y_0) + F_z(x_0,y_0,z_0)(z - z_0) = 0$$

S17 #6 Find $\lim\limits_{(x,y)\to(0,0)} \frac{3x - 2y}{\sqrt{x^2+y^2}}$

$$\lim_{(x,y)\to(0,0)} \frac{e^{x+y}}{1+e^{x-y}}$$

In the second $\frac{e^{0+0}}{1+e^{0-0}} = \frac{1}{1+1} = \frac{1}{2}$ is a number

$$\boxed{\text{so } \lim_{(x,y)\to(0,0)} \frac{e^{x+y}}{1+e^{x-y}} = \frac{1}{2}}$$

$$\boxed{\text{so} \lim_{(x,y)\to(0,0)} \frac{e^{\cdot}\ \cdot}{1+e^{x-y}} = \frac{1}{2}}$$

In the first setting $y=0$, we get

$$\lim_{x\to 0} \frac{3x}{\sqrt{x^2}} = \lim_{x\to 0} \frac{3x}{|x|} \text{ DNE}$$

$$\boxed{\text{so} \lim_{(x,y)\to(0,0)} \frac{3x-2y}{\sqrt{x^2+y^2}} \text{ DNE}}$$

F16 #9, $P = \sqrt{u^2+v^2+w^2}$

$u = u(x,y)$ $\quad u(0,1) = 0$

$v = v(x,y)$ $\quad u_x(0,1) = 2$

$w = w(x,y)$ $\quad w(0,1) = 2$

$w_x(0,1) = 0$

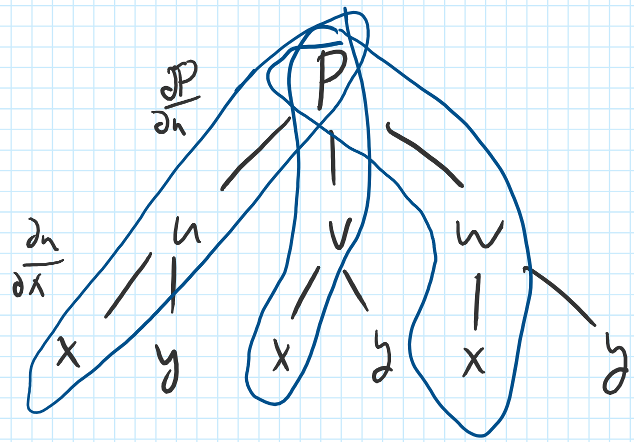
$v(x,y) = ye^x$

$v(0,1) = 1$

$v_x = ye^x$

$v_x(0,1) = 1$

Find $P_x(0,1)$



$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial P}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial P}{\partial w}\cdot\frac{\partial w}{\partial x}$$

$$\frac{\partial P}{\partial u} = \frac{1}{2\sqrt{u^2+v^2+w^2}}\cdot 2u = \frac{u}{\sqrt{u^2+v^2+w^2}}$$

Similar $\frac{\partial P}{\partial v} = \frac{v}{\sqrt{u^2+v^2+w^2}}$, $\frac{\partial P}{\partial w} = \frac{w}{\sqrt{u^2+v^2+w^2}}$

at $(0,1)$

$$\frac{\partial P}{\partial u}(0,1) = \frac{0}{\sqrt{0^2+1^2+2^2}} = 0$$

$$\frac{\partial P}{\partial v}(0,1) = \frac{1}{\sqrt{0^2+1^2+2^2}} = \frac{1}{\sqrt{5}}$$

$$\frac{\partial P}{\partial w}(0,1) = \frac{2}{\sqrt{0^2+1^2+2^2}} = \frac{2}{\sqrt{5}}$$

Answer,

$\partial P\ (0,1) = \ldots$

Answer,
$$\frac{\partial P}{\partial x}(0,1) = 0 \cdot 2 + \frac{1}{\sqrt{5}} \cdot 1 + \frac{2}{\sqrt{5}} \cdot 0 = \frac{1}{\sqrt{5}}$$

or $5^{-1/2}$ D