261-2

Monday, August 20, 2018 1:32 PM

Math 261, Lecture 2, 8/22/2018

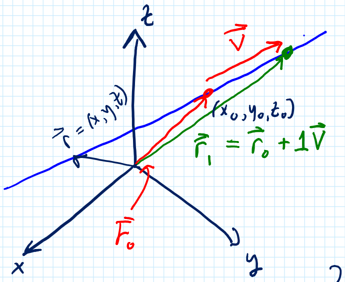
Announcements
- HW 1,2 Due 11pm Thurs.
- Cengage account
- HW appeals
  homework.appeal.webassign@purdue.edu

Outline. §12.5 begin

§12.5 Equations of Lines and Planes

Line = point + direction

$\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$ $\quad \vec{v} = \langle a, b, c \rangle$



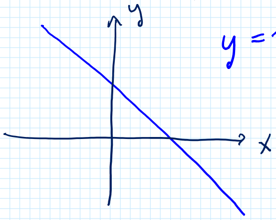
Parametric Equation of Line

$$\underline{\vec{r}} = \vec{r}_0 + \underline{t}\,\vec{v}$$

Rewrite using $\vec{r} = \langle x, y, z \rangle$

$$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$$
$$= \langle x_0, y_0, z_0 \rangle + \langle at, bt, ct \rangle$$
$$= \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$$

2D case



$y = mx + b$

$$\begin{cases} y = mt + b \\ x = t \end{cases}$$

Parametric form
$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$
given in terms of a parameter "t"

Ex. Find parametric eqn of line that passes thru $(1,4,-3)$ parallel to $5i + j - 7k$

$\vec{r}_0 = \langle 1, 4, -3 \rangle$, $\vec{v} = \langle 5, 1, -7 \rangle$

$\vec{r} = \langle 1,4,-3 \rangle + t\langle 5, 1, -7 \rangle$

$= \langle 1 + 5t, 4 + t, -3 - 7t \rangle$

Sol'n
$x = 1 + 5t$
$y = 4 + t$
$z = -3 - 7t$

Ex. Find a line thru $(1,4,-3)$ $\perp$ to $5i + j - 7k$

$\langle c, d, e \rangle = \vec{w} \perp \langle 5, 1, -7 \rangle$,

$\langle c, d, e \rangle \cdot \langle 5, 1, -7 \rangle = 0$

$\to 5c + d - 7e = 0$

$\vec{w} = \langle 1, 2, 1 \rangle$ for instance

$\vec{r} = \langle 1, 4, -3 \rangle + t\langle 1, 2, 1 \rangle$

$\vec{w} = \langle 1, 2, 1 \rangle$

another $\langle 0, 7, 1 \rangle, \langle 7, 0, 5 \rangle$

Symmetric Equation of a Line

Line thru $(1, 2, -1)$ parallel to $\langle 2, 4, 5 \rangle$

Parametric
$$\begin{cases} x = 1 + 2t \\ y = 2 + 4t \\ z = -1 + 5t \end{cases}$$

Solve for t

$2t = x - 1$ $\quad t = \frac{x-1}{2}$

$4t = y - 2$ $\quad t = \frac{y-2}{4}$

$5t = z + 1$ $\quad t = \frac{z+1}{5}$

$$\boxed{\text{Symmetric form} \quad \frac{x-1}{2} = \frac{y-2}{4} = \frac{z+1}{5}}$$

$\vec{r} = \underset{\vec{r}_0}{\langle x_0, y_0, z_0 \rangle} + t\underset{\vec{v}}{\langle a, b, c \rangle}$

symmetric form is

$\vec{r} = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$

$\vec{r_0}$ $\vec{v}$

symmetric form is

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}, \quad a, b, c \neq 0$$

$$a = 0, \quad x = x_0 \quad \frac{y - y_0}{b} = \frac{z - z_0}{c} \quad b, c \neq 0$$

other cases are similar

Ex. Line segment $\overset{P}{(0, 1, -1)}$ to $\overset{Q}{(2, -3, 1)}$

Where does the line intersect xz plane?

Line is $\vec{r} = \langle 0, 1, -1 \rangle + t\langle 2, -4, 2 \rangle$

or $\begin{cases} x = 2t \\ y = 1 - 4t \\ z = -1 + 2t \end{cases}$

xz plane is $y = 0$

$0 = y = 1 - 4t$

$t = 1/4$

so $x = 2(1/4) = 1/2$

$y = 1 - 4(1/4) = 0$

$z = -1 + 2(1/4) = -1/2$

thus segment intersects xz plane at $(1/2, 0, -1/2)$

$P = \vec{r_0} = \langle 0, 1, -1 \rangle$, direction from P to Q

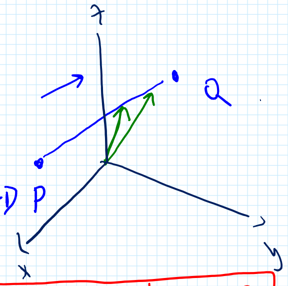
$\vec{PQ} = Q - P$

$= \langle 2, -3, 1 \rangle - \langle 0, 1, -1 \rangle$ P

$= \langle 2, -4, 2 \rangle$

$\vec{r} = \langle 0, 1, -1 \rangle + t\langle 2, -4, 2 \rangle$

$= P + t(Q - P)$



$t = 0 \quad P$

$Q \overset{?}{=} P + t(Q - P), \quad t = 1$

Line segment P to Q

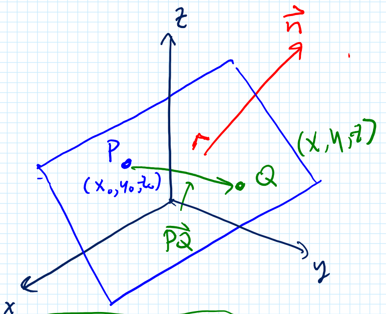
$P + t\,\vec{PQ} \quad 0 \le t \le 1$

Ex. Line that doesn't intersect xz plane?

Equations of Planes

Plane = point + normal

$\vec{r_0} = \langle x_0, y_0, z_0 \rangle \qquad \vec{n} = \langle a, b, c \rangle$



$\vec{PQ} \perp \vec{n}$

$\vec{n} \cdot \vec{PQ} = 0$

$\vec{PQ} = Q - P = \langle x - x_0, y - y_0, z - z_0 \rangle$

$\vec{n} \cdot \vec{PQ} = \boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$

Rewrite as scalar eqn

$5(x - 1) + 6(y + 3) - (z + 4) = 0$

$\vec{n} = \langle 5, 6, -1 \rangle$

$(x_0, y_0, z_0) = \langle 1, -3, -4 \rangle$

algebra $5x - 5 + 6y + 18 - z - 4 = 0$

$5x + 6y - z + 9 = 0$

General equation for a Plane

$$\boxed{ax + by + cz + d = 0}$$

$\vec{n} = \langle a, b, c \rangle$,

set two of $x, y, z = 0$
solve the last to find a point

Ex. Find scalar eqn of plane and sketch

$$4x - 3y + 6z - 12 = 0$$

$\vec{n} = \langle 4, -3, 6 \rangle$

$\vec{r}_0 = \langle 0, 0, 2 \rangle$, $\langle 3, 0, 0 \rangle$

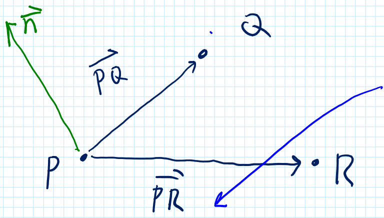
$x = y = 0$
$6z - 12 = 0$
$z = 12/6 = 2$

or
$y = z = 0$
$4x - 12 = 0$
$x = 3$

$$\left.\begin{aligned} \vec{n}\cdot(\vec{r} - \vec{r}_0) &= 0 \\ 4x - 3y + 6(z-2) &= 0 \\ \text{or } 4(x-3) - 3y + 6z &= 0 \end{aligned}\right\} \text{ same plane}$$

Ex. Find the plane that contains the three points

$P = (1, 2, -1)$ $Q = (1, 3, 3)$ $R = (2, 4, -1)$



$$\begin{aligned} \vec{PQ} &= \langle 1,3,3 \rangle - \langle 1,2,-1 \rangle \\ &= \langle 0,1,4 \rangle \\ \vec{PR} &= \langle 2,4,-1 \rangle - \langle 1,2,-1 \rangle \\ &= \langle 1,2,0 \rangle \end{aligned}$$

$$\left[\begin{aligned} \vec{n} &= \vec{PQ} \times \vec{PR} = \langle 0,1,4 \rangle \times \langle 1,2,0 \rangle \\ \vec{r}_0 &= P = \langle 1, 2, -1 \rangle \end{aligned}\right.$$

$$\vec{n} = \langle 0,1,4 \rangle \times \langle 1,2,0 \rangle = \begin{vmatrix} i & j & k \\ 0 & 1 & 4 \\ 1 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} i - \begin{vmatrix} 0 & 4 \\ 1 & 0 \end{vmatrix} j + \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} k$$

$$= (1\cdot 0 - 4\cdot 2)i - (0\cdot 0 - 4\cdot 1)j + (0\cdot 2 - 1\cdot 1)k$$

$$= -8i + 4j - k$$

Scalar equation $\vec{n}\cdot(\vec{r} - \vec{r}_0)$

$$\boxed{\text{sol'n} \quad -8(x-1) + 4(y-2) - (z+1) = 0}$$