261-20

Wednesday, October 10, 2018 1:23 PM

Math 261, Lecture 20, 10/10/18

EXAM 1 - Avg 75%

- A ≥ 90%
- B ≥ 75%
- C ≥ 60%
- D ≥ 50%

* EXAM 2 is in 3 weeks from today

Today §15.1 (all), Next: §15.2 (all)

Recap: §14.8 Lagrange Multipliers

Goal: Minimize/Maximize a function $f(x,y)$

Given a constraint curve $g(x,y)=c$

Solve the system $\begin{cases} \vec{\nabla} f(x,y) = \lambda \vec{\nabla} g(x,y) \\ g(x,y) = c \end{cases}$

$$= \begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g = c \end{cases}$$

Steps: 1) use an equation to solve for a variable

2) substitute in to a second equation

3) repeat until you reduced to one variable.

* Try not to use same equ'n twice.

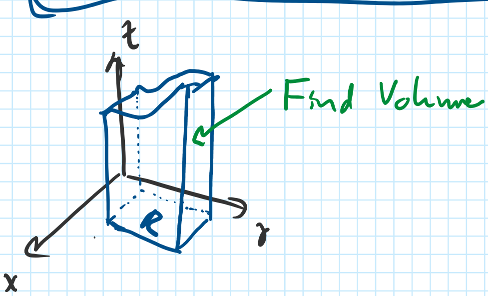
Chapter 15 - Multiple Integrals

§15.1 Double Integrals over Rectangles

$R = [a,b] \times [c,d]$

xy-plane

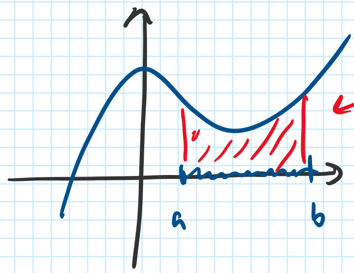
$z = f(x,y)$ above



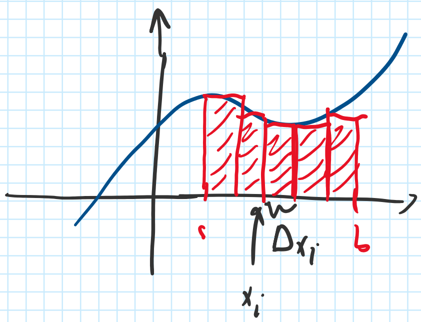
$z = f(x,y)$ above the xy-plane on R. (positive z-values)

1st Goal: Find volume of solid bounded above by $f(x,y)$ and below by xy-plane over R.

Calc Review



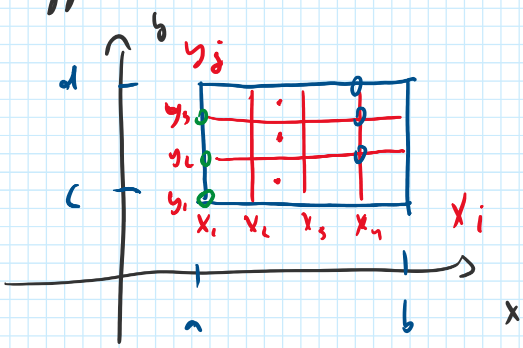
Find Area $= \int_a^b f(x)\,dx$



Riemann sum

$$\sum_i^5 f(x_i)\,\Delta x_i \xrightarrow[\Delta x_i \to 0]{\text{limit}} \int_a^b f(x)\,dx$$

What happens in two variables



$R = [a,b] \times [c,d]$

$$= \left\{ (x,y) : \begin{array}{l} a \le x \le b \\ c \le y \le d \end{array} \right.$$

5 possibilities for test pts

lower left corner, upper right corner, midpoint, etc.

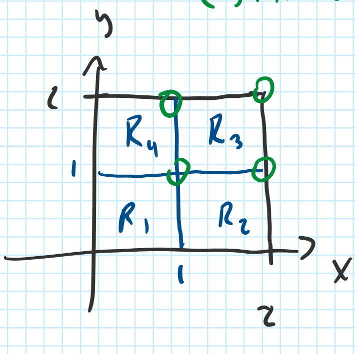
$$V \approx \sum_j \sum_i f(x_{ij}, y_{ij}) \underbrace{\Delta x_i \Delta y_j}_{\Delta A}$$

"area of small rectangle"

2-variable Riemann sum.

Ex. $z = 16 - 3x^2 + 5y^2 \qquad R = [0,2] \times [0,2]$

Estimate using upper right corners 4 equal subrectangles



test pts $(1,1), (1,2), (2,1), (2,2)$

$$V \approx \sum_{j=1}^{2} \sum_{i=1}^{2} f(x_{ij}, y_{ij})\, \Delta A$$

$dA = \text{area } R_i = 1$

$$V \approx f(1,1)\cdot 1 + f(1,2)\cdot 1 + f(2,2)\cdot 1 + f(2,1)\cdot 1$$

$$18\cdot 1 + 33\cdot 1 + 24\cdot 1 + 9\cdot 1$$

$$= 84$$

Ex. by Midpoints instead $\quad R_1 - (1/2, 1/2), \quad R_3 = (3/2, 3/2)$

$R_2 - (3/2, 1/2), \quad R_4 = (1/2, 3/2)$

$$V \approx f(\tfrac{1}{2}, \tfrac{1}{2})\cdot 1 + f(\tfrac{3}{2}, 1/2)\cdot 1 + f(3/2, 3/2)\cdot 1 + f(1/2, 3/2)\cdot 1$$

$$= 16.5 + 10.5 + 20.5 + 26.5 = 74$$

Hard to evaluate volumes as limits of Riemann sums!

Double Integral.

$$\int_c^d \int_a^b f(x,y)\,dx\,dy \quad \text{on } R = [a,b] \times [c,d]$$

$$\int_{y=c}\int_{x=a} f(x,y)\,dx\,dy \quad \text{on} \quad R=[a,b]\times[c,d]$$

integrate from inside out

- y is constant, integrate in x
- left with a function in y, which we can integrate

Fubini's Thm – $V=\int_c^d\int_a^b f(x,y)\,dx\,dy=\int_a^b\int_c^d f(x,y)\,dy\,dx$

Ex. $$\int_{y=1}^{2}\int_{x=0}^{1} x^2+3xy+y^2\,dx\,dy$$

$$\int_{y=1}^{2}\left[\frac{1}{3}x^3+\frac{3}{2}x^2y+y^2x\right]\Bigg|_{x=0}^{1}\,dy$$

$$=\int_{y=1}^{2}\frac{1}{3}+\frac{3}{2}y+y^2\,dy$$

$$=\left[\frac{1}{3}y+\frac{3}{4}y^2+\frac{1}{3}y^3\right]\Bigg|_{y=1}^{2}=\frac{1}{3}+\frac{9}{4}+\frac{7}{3}$$

$$=\frac{59}{12}\quad \text{Answer}$$

2 4

Ex. $\displaystyle\int_1^2\int_3^4 \frac{xy}{(x^2+y^2)^3}\,dx\,dy$

$$\int_1^2 y\left[\int_3^4 \frac{x}{(x^2+y^2)^3}\,dx\right]dy$$

$u = x^2+y^2$

$du = 2x\,dx$ or $\frac{1}{2}du = x\,dx$

$$\int_1^2 y\left[-\frac{1}{4}\cdot\frac{1}{(x^2+y^2)^2}\right]\Bigg|_{x=3}^{4} dy$$

$$= \int_1^2 \frac{-y}{4(16+y^2)^2} + \frac{y}{4(9+y^2)^2}\,dy$$

$$= \frac{1}{8}\cdot\frac{1}{16+y^2} - \frac{1}{8}\,\frac{1}{9+y^2}\Bigg|_{y=1}^{2} = \frac{1}{8}\left[\frac{1}{20}-\frac{1}{18}-\frac{1}{13}+\frac{1}{10}\right]$$

$$= \frac{1}{8}\left[\frac{117-130-180+234}{2340}\right]$$

$$= \frac{1}{8}\left(\frac{41}{2340}\right) = \frac{41}{18720}$$

Ex. $\displaystyle\int_0^1\int_0^3 \frac{y}{e^{xy}}\,dy\,dx$

to evaluate $\displaystyle\int_0^3 \frac{y}{e^{xy}}\,dy$ we need to use integration by parts

OR switch integrands

$\displaystyle\int_0^1\int_0^3 \frac{y}{\;}$ ... $\displaystyle\int_0^3\int_0^1 \frac{y}{\;}$ ...

$$\int_0^1 \int_0^3 \frac{y}{e^{xy}}\, dy\, dx = \underbrace{\int_0^3 \int_0^1 \frac{y}{e^{xy}}\, dx\, dy}$$ by Fubini

$$\int_0^3 \int_0^1 y e^{-xy}\, dx\, dy = \int_0^3 \left[-e^{-xy}\right]\Big|_{x=0}^{1} dy$$

$u = yx$

$du = y\, dx$

$$= \int_0^3 (-e^{-y} + 1)\, dy$$

$$= \left[e^{-y} + y\right]\Big|_{y=0}^{3} = e^{-3} - 1 + 3 - 0$$

$$= 2 + e^{-3}$$

Answer