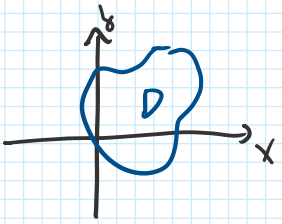
Math 261, Lecture 24, 10/19/18

Today: §15.6, Next: §15.7

Recap:



$\rho(x,y)$ density on D

m = mass $= \iint_D \rho(x,y)\,dA$

Center of mass $(\bar{x},\bar{y})$

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\,\rho(x,y)\,dA$$

$$\bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\,\rho(x,y)\,dA$$

Surface area $z = f(x,y)$ above region D

$$SA = \iint_D \sqrt{(f_x)^2 + (f_y)^2 + 1}\;dA$$

$y = f(x)$

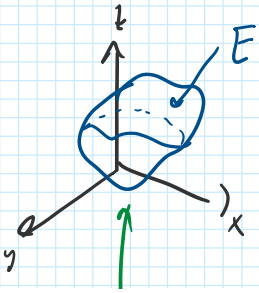
$$\int_a^b \sqrt{1+(f')^2}\,dx$$

Arc Length

$F(x,y,z) = z - f(x,y) = 0$

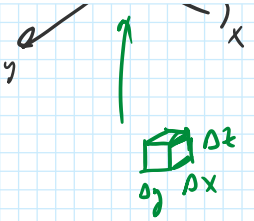
$$SA = \iint_D |\nabla F|\,dA$$

§15.6 Triple Integrals



$f(x,y,z)$

$$\iiint f(x,y,z)\,dV$$



$$\iiint_E f(x,y,z)\, dV$$

$$dV = dx\, dy\, dz$$

or $dz\, dx\, dy$

or $dy\, dz\, dx$

$$\text{Volume of } E = \iiint_E 1\, dV$$

Ex. $R = [a,b] \times [c,d] \times [r,s]$

$$\begin{cases} a \le x \le b \\ c \le y \le d \\ r \le z \le s \end{cases}$$

$$f(x,y,z) = xyz$$

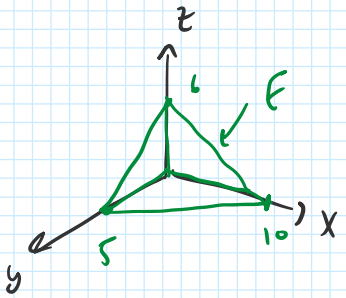
$$\int_r^s \int_c^d \int_a^b xyz \; dx\, dy\, dz$$

$$= \int_r^s \int_c^d \frac{1}{2} x^2 yz \Big|_{x=a}^{b} dy\, dz$$

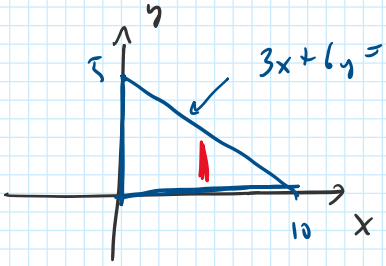
$$= \int_r^s \int_c^d \frac{1}{2}(b^2 - a^2)\, yz \; dy\, dz$$

...

Ex. $f(x,y,z) = x^2 y$ Integrate over solid given by plane $3x + 6y + 5z = 30$ in first octant $x \geq 0, y \geq 0, z \geq 0$



$$\iiint_E x^2 y \, dV$$



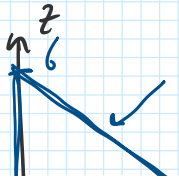
$3x + 6y = 30 \rightsquigarrow y = -\frac{1}{2}x + 5$

$$\int_{x=0}^{10} \int_{y=0}^{y=-\frac{1}{2}x+5} \int_{z=0}^{z=6-\frac{3}{5}x-\frac{6}{5}y} x^2 y \, dz \, dy \, dx$$

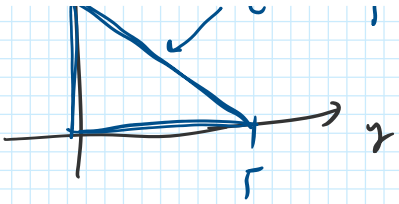
$$= \int_{x=0}^{10} \int_{y=0}^{-\frac{1}{2}x+5} \left[x^2 y z \Big|_{z=0}^{6-\frac{3}{5}x-\frac{6}{5}y} \right] dy \, dx$$

$$= \int_{x=0}^{10} \int_{y=0}^{-\frac{1}{2}x+5} x^2 y \left(6 - \frac{3}{5}x - \frac{6}{5}y\right) dy \, dx$$

back is yz plane



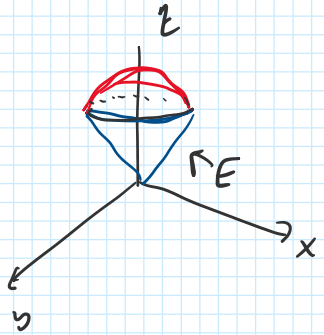
$z = -\frac{6}{5}y + 6$



$$\int_{y=0}^{5}\int_{z=0}^{-\frac{6}{5}y+6}\int_{x=0}^{10-2y-\frac{5}{3}z} x^2 y \; dx\, dz\, dy$$

$$= \int_{y=0}^{5}\int_{z=0}^{-\frac{6}{5}y+6} \frac{1}{3}y\left(10-2y-\frac{5}{3}z\right)^3 dz\, dy$$

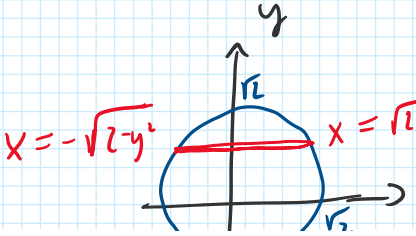
Ex. Find volume of the region $x^2+y^2+z^2 = 4$, $x^2+y^2-z^2 = 0$, $z \geq 0$



$$\text{Volume} = \iiint_E 1 \; dV$$

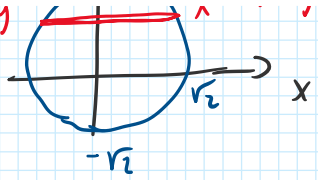
$$\begin{cases} x^2+y^2+z^2 = 4 \\ z^2 = x^2+y^2 \end{cases} \rightsquigarrow 2x^2+2y^2 = 4$$

$$x^2+y^2 = 2$$

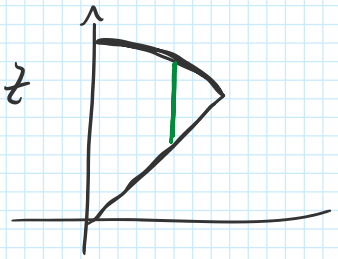


$$\int_{\sqrt{2}} \int_{\sqrt{2-y^2}} \int_{z=\sqrt{4-x^2-y^2}} 1 \; dz\, dx\, dy$$

$x = -\sqrt{2-y^2}$



$$\int_{y=-\sqrt{2}}^{\sqrt{2}} \int_{x=-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{z=\sqrt{x^2+y^2}}^{1} 1 \; dz\, dx\, dy$$



$z^2 = x^2 + y^2$

$z = \sqrt{x^2+y^2}$