Exam 2 10/31 (Wed) 6:30 pm

Elliott Hall

14.7 - 16.1

Recap Triple Integrals

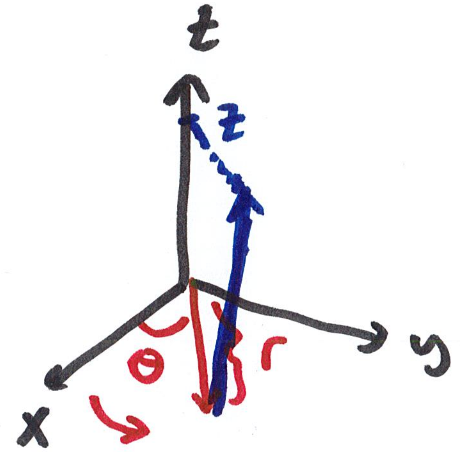
$$dV = dx\,dy\,dz$$

$$\iiint_E 1 \, dV = \text{Volume of } E$$

$$E = [a,b] \times [c,d] \times [r,s]$$

$$\int_r^s \int_c^d \int_a^b f(x,y,z)\, dx\,dy\,dz$$

§ 15.7 Cylindrical Coords



$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$r^2 = x^2 + y^2$

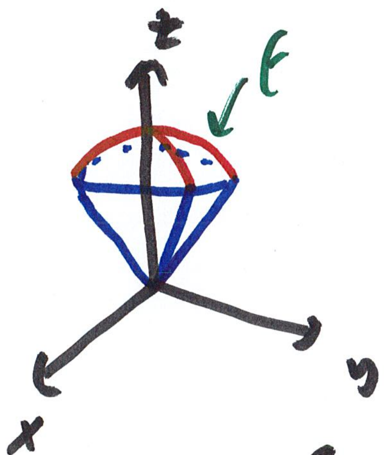
$\tan\theta = y/x$

$$dV = r\,dr\,d\theta\,dz$$

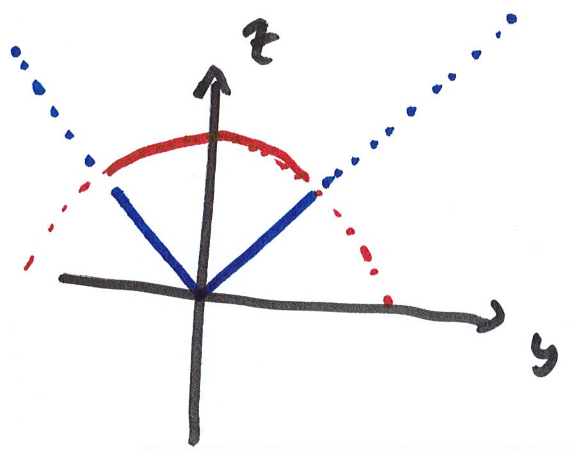
Ex. intersection of $x^2+y^2+z^2=4$

with cone $x^2+y^2=z^2$

$z \geq 0$



Find volume



$$\iiint_E 1 \, dV$$

Last time...

$$\int_{y=-\sqrt{2}}^{\sqrt{2}} \int_{x=-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{z=\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} 1 \, dz \, dx \, dy$$

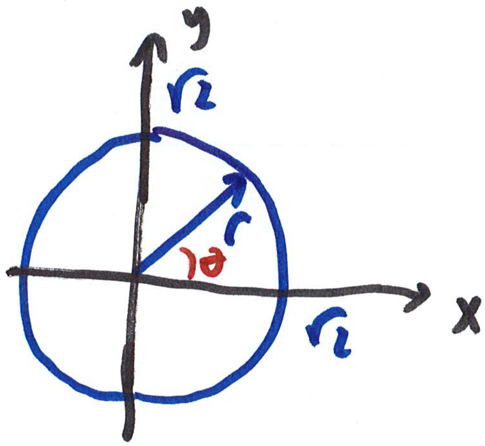
$$x^2+y^2+z^2=4$$

$$x^2+y^2=z^2$$

$\theta \to r \to z$

$$2x^2+2y^2=4$$

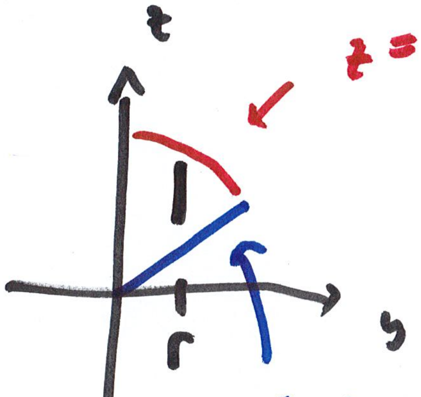
$$x^2+y^2=2$$



$0 \le r \le \sqrt{2}$

$0 \le \theta \le 2\pi$

$z=\sqrt{4-x^2-y^2}=\sqrt{4-r^2}$



$z=\sqrt{x^2+y^2}=\sqrt{r^2}=r$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{2}}\int_{z=r}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{2}} \left[r z \Big|_{z=r}^{\sqrt{4-r^2}} \right] dr\, d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{2}} r\sqrt{4-r^2} - r^2 \, dr\, d\theta$$

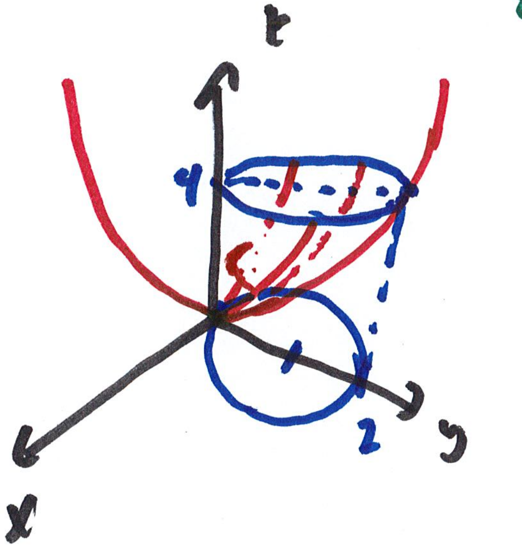
$$= \int_{\theta=0}^{2\pi} -\frac{1}{3}(4-r^2)^{3/2} - \frac{1}{3} r^3 \Big|_{r=0}^{\sqrt{2}} d\theta$$

Ex. Volume of solid contained by

$z = x^2 + y^2$ (paraboloid)

$x^2 + y^2 = 2y$ (cylinder)

$z = 4$

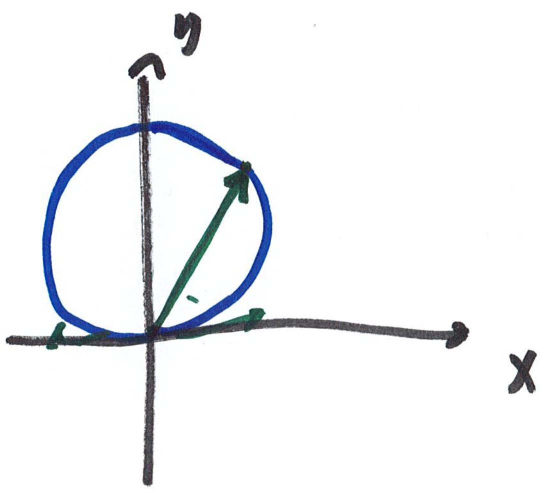


$x^2 + y^2 - 2y + 1 = 0 + 1$

$x^2 + (y-1)^2 = 1$

radius $= 1$

center $(0,1)$



$x^2 + y^2 = 2y$

$r^2 = 2r\sin\theta$

$r = 2\sin\theta$

$0 \le \theta \le \pi$

$0 \le r \le 2\sin\theta$

$\cancel{z = r^2}$ $\quad r^2 \le z \le 4$

$$\int_{\theta=0}^{\pi}\int_{r=0}^{2\sin\theta}\int_{r^2}^{4} r \, dz\, dr\, d\theta = r(4-r^2)$$