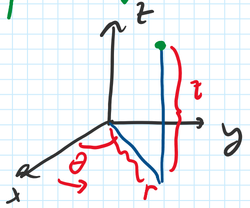
Math 261, Lecture 26, 10/24/18

- EXAM 2 WED 10/31 6:30PM §§ 14.7, 14.8, 15.1–15.8, 16.1
  ELLIOTT HALL

- Flu Shot :)

Today: §15.8, Next: §16.1

Recap: Cylindrical Coordinates



$(x,y,z) \rightsquigarrow (r,\theta,z)$

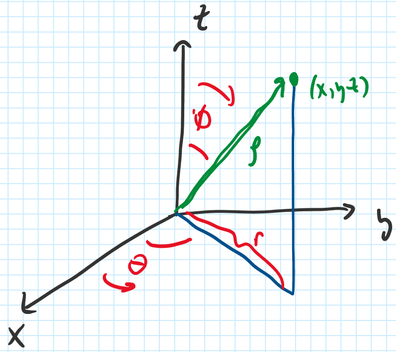
$x = r\cos\theta$
$y = r\sin\theta$
$z = z$
$x^2+y^2 = r^2$

$\theta \downarrow r \downarrow z$ $\quad dz\,dr\,d\theta$

$dV = r\,dr\,d\theta\,dz$

Use for rotation about z-axis, cylinders, cones, spheres...

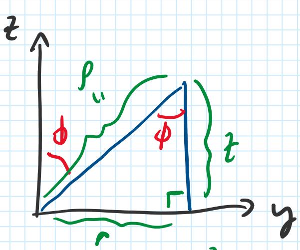
§15.8 Spherical Coordinates



ϕ baby phi $\quad \Phi$

spherical coordinates
(ρ, θ, ϕ)

$\rho \geq 0, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \phi \leq \pi$



$$\begin{cases} \rho^2 = x^2+y^2+z^2 \\ z = \rho\cos\phi \\ x = r\cos\theta = \rho\sin\phi\cos\theta \\ y = r\sin\theta = \rho\sin\phi\sin\theta \end{cases}$$

$\cos\phi = \frac{z}{\rho}$

$$\begin{cases} z \\ x = r\cos\theta = \rho\sin\phi\cos\theta \\ y = r\sin\theta = \rho\sin\phi\sin\theta \\ r = \rho\sin\phi \end{cases}$$

$\cos\phi = \dfrac{z}{\rho}$

$$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi \qquad \phi \to \theta \to \rho$$

$$= \rho \, d\phi \cdot \underbrace{r \, d\theta}_{\rho \sin\phi \, d\theta} \cdot d\rho$$

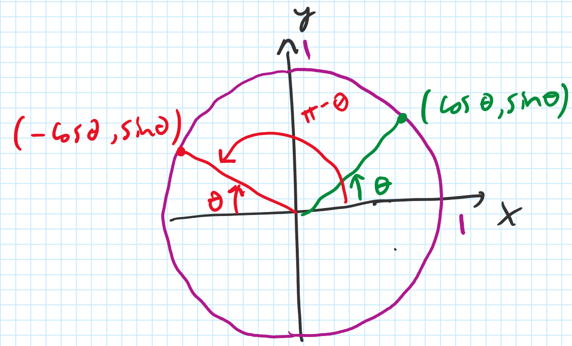
Ex. Convert $(1,-1,-\sqrt{2})$ to spherical coordinates

$$\rho = \sqrt{1^2 + (-1)^2 + (-\sqrt{2})^2} = \sqrt{1+1+2} = \sqrt{4} = 2$$

$$\boxed{\rho = 2}$$

$$\cos\phi = \frac{z}{\rho} = \frac{-\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$$

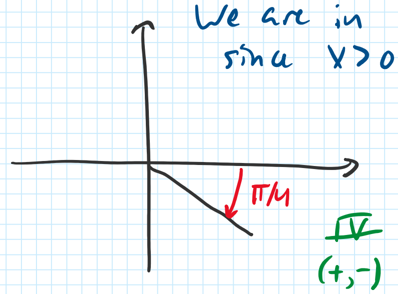
$$\phi = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$



$$\tan\theta = \frac{y}{x} = -1$$

We are in quadrant IV since $x > 0$ and $y < 0$.

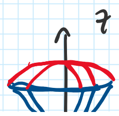
$$\boxed{\left(2, \frac{7\pi}{4}, \frac{3\pi}{4}\right)}$$

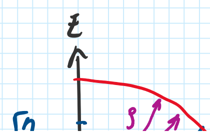


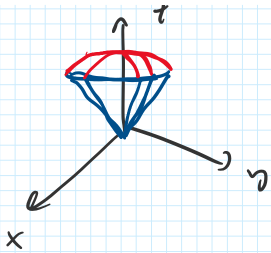
Ex. $x^2 + y^2 + z^2 = 4$

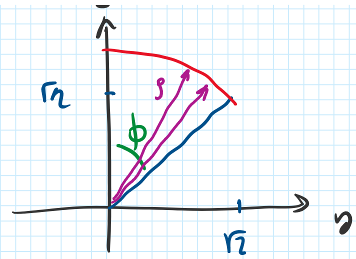
$x^2 + y^2 = z^2, \quad z \geq 0$

Find volume using spherical.









$0 \le \phi \le \pi/4$, $0 \le \rho \le 2$, $0 \le \theta \le 2\pi$

$$\iiint_E 1\, dV = \int_0^{\pi/4}\int_0^{2\pi}\int_0^{2} 1\, \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi = \int_0^{\pi/4}\int_0^{2\pi}\left[\frac{1}{3}\rho^3\Big|_0^2\right]\sin\phi\, d\theta\, d\phi$$

$$= \frac{16\pi}{3}\int_0^{\pi/4}\sin\phi\, d\phi = \frac{16\pi}{3}\left[-\cos\phi\Big|_{\phi=0}^{\pi/4}\right]$$

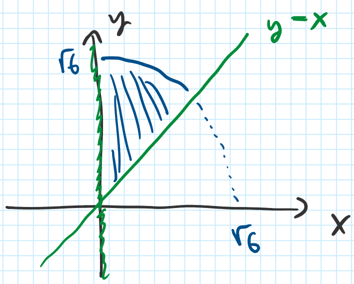
$$= \frac{16\pi}{3}\left[1 - \frac{\sqrt{2}}{2}\right]$$

Ex. $y = \sqrt{6 - x^2 - z^2}$, $x = 0$, $y = x$

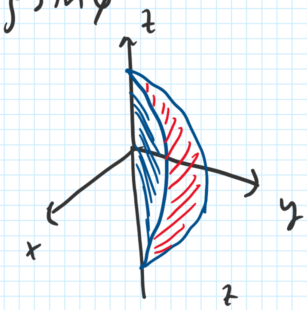
E region contained by these

integrate distance from z-axis.

distance from z-axis $= \sqrt{x^2+y^2} = r = \rho\sin\phi$



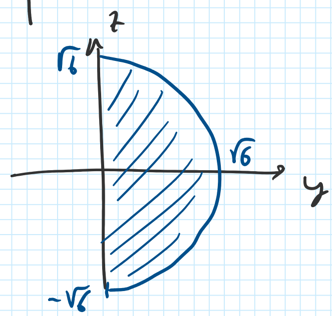
$\frac{1}{8}$ wedge of sphere



$0 \le \rho \le \sqrt{6}$

$0 \le \phi \le \pi$

$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$



$$\int_{\phi=0}^{\pi}\int_{\theta=\pi/4}^{\pi/2}\int_0^{\sqrt{6}} \rho\sin\phi \cdot \rho^2\sin\phi \, d\rho\, d\theta\, d\phi$$

2

$$= \int_{\phi=0}^{\pi}\int_{\theta=\pi/4}^{\pi/2}\left[\frac{1}{4}\rho^4\Big|_0^{\sqrt{6}}\right]\sin^2\phi\, d\theta\, d\phi$$

π

...

$$= \frac{9\pi}{4} \int_{\phi=0}^{\pi} \sin^2\phi \, d\phi = \frac{9\pi}{4} \int_0 \frac{1-\cos(2\phi)}{2} \, d\phi$$

$$= \frac{9\pi}{8} \left[\phi - \frac{\sin(2\phi)}{2} \right] \Big|_0^{\pi}$$

$$= \frac{9\pi^2}{8}$$