Math 261, Lecture 27, 10/26/18

- EXAM 2, Study Guide and Seating Available

- Office Hours for Next Week: MATH 744
  - M, 3:30 - 5:30
  - T, 2:30 - 4:30
  - W, 3:30 - 5:30
  - F, canceled

Today §16.1, Next §16.2

Recap. Spherical (ρ, θ, ϕ)

$$x = \rho\sin(\phi)\cos(\theta)$$
$$y = \rho\sin(\phi)\sin(\theta)$$
$$z = \rho\cos(\phi)$$
$$\rightarrow r = \rho\sin(\phi)$$

$$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

* θ is same as θ in polar/cylindrical

* Note how r appears "in disguise" in the formulas for x and y.

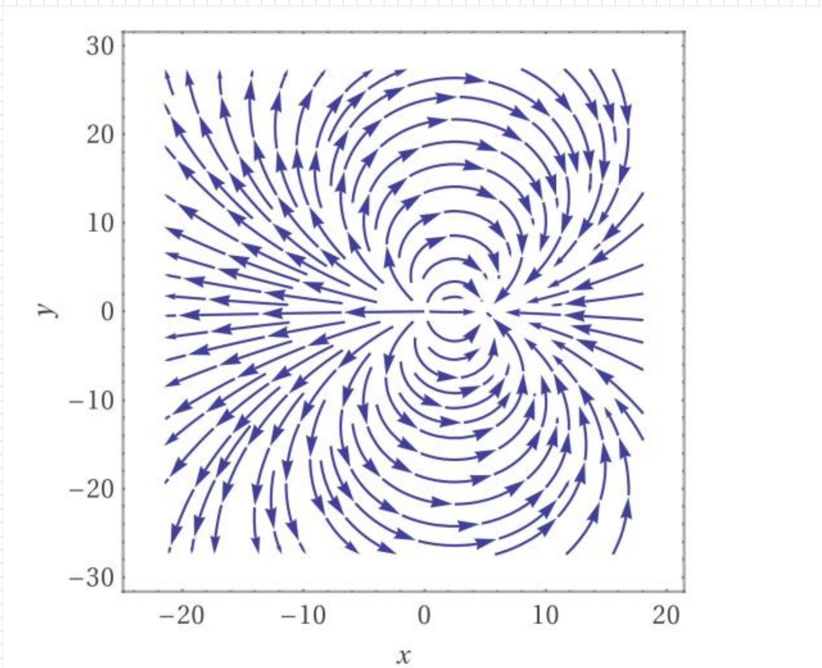
§16.1 Vector Fields

A vector field is function

$F(x,y)$ which outputs a 2D vector

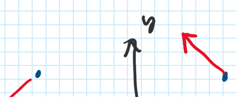
or

$F(x,y,z)$ which outputs a 3D vector.



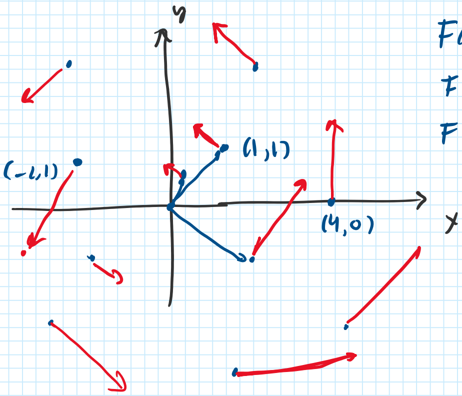
Magnetic dipole

Ex. $F(x,y) = -y\,i + x\,j$
$= \langle -y, x \rangle$



$F(1,1) = \langle -1, 1 \rangle$

$F(4,0) = \langle 0, 4 \rangle$



$F(1,1) = \langle -1, 1 \rangle$

$F(4,0) = \langle 0, 4 \rangle$

$F(-2,1) = \langle -1, -2 \rangle$

x

Magnetic dipole

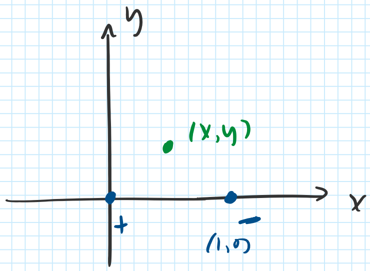
$$\langle -y, x \rangle \cdot \langle x, y \rangle = -yx + xy = 0$$

$$F(r\cos\theta, r\sin\theta) = \langle -r\sin\theta, r\cos\theta \rangle$$

$$F = \frac{\partial}{\partial\theta}$$

Ex. (Magnetic Dipole)

$$f(x,y) = \frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\sqrt{(x-1)^2+y^2}}$$



strength of attraction/repulsion proportional to distance from pole

$$F(x,y) = \vec{\nabla} f(x,y) = \langle f_x, f_y \rangle$$

↑ F has potential f if $F = \vec{\nabla} f$

$$F(x,y) = \left\langle -\frac{1}{2}\frac{2x}{(x^2+y^2)^{3/2}} - \left(-\frac{1}{2}\frac{2(x-1)}{((x-1)^2+y^2)^{3/2}}\right), \frac{-y}{(x^2+y^2)^{3/2}} - \frac{-y}{((x-1)^2+y^2)^{3/2}} \right\rangle$$

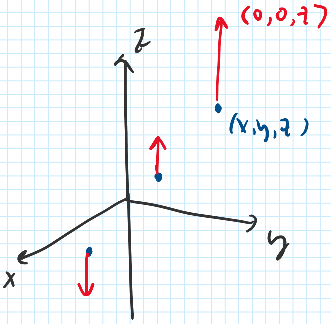
$$= \left\langle \frac{-x}{(x^2+y^2)^{3/2}} + \frac{(x-1)}{((x-1)^2+y^2)^{3/2}}, \frac{-y}{(x^2+y^2)^{3/2}} + \frac{y}{((x-1)^2+y^2)^{3/2}} \right\rangle$$

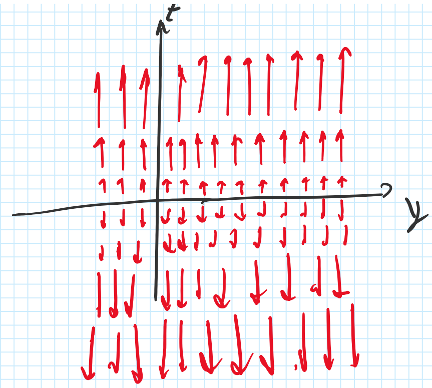
$|F(x,y)|$ magnitude, $G(x,y) = \dfrac{F(x,y)}{|F(x,y)|}$ vector field of directional components (unit vectors)

↑ This is the vector field graphed above for the magnetic dipole.

Ex. Plot $F(x,y,z) = z\,k$

graphs above for the magnetic dipole.





Ex. $F(x,y,z) = y\,i + z\,j + x\,k = (y, z, x)$

The map F exchanges (rotates) coordinate axes