# Math 261, Lecture 3, 8/24/18

**Outline** §12.5 finish, §12.6 begin Quadric Surfaces

## §12.5 Equations of Lines and Planes (cont'd)

Recap: → Line thru $(x_0, y_0, z_0)$ parallel to $\vec{v} = \langle a, b, c \rangle$

$$\vec{r} = \langle x_0 + at,\ y_0 + bt,\ z_0 + ct \rangle$$

→ Plane thru $(x_0, y_0, z_0)$ $\perp$ to $\vec{n} = \langle a, b, c \rangle$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$\boxed{\text{All } (x, y, z) \text{ that satisfy}}$

Ex. Where does the line $\langle \overset{x}{1 - 2t}, \overset{y}{-1 + 3t}, \overset{z}{3 - 4t} \rangle$ intersect the plane $x + 2y - z + 5 = 0$ ?

Parametric eqn of line
$$x = 1 - 2t$$
$$y = -1 + 3t$$
$$z = 3 - 4t$$

Point in plane must also satisfy plane equation

Substitute into general eqn of plane

$$\underbrace{(1 - 2t)}_{x} + 2\underbrace{(-1 + 3t)}_{y} - \underbrace{(3 - 4t)}_{z} + 5 = 0$$

$$1 - 2t - 2 + 6t - 3 + 4t + 5 = 0$$

$$-2t + 6t + 4t + 1 - 2 - 3 + 5 = 0$$

$$8t + 1 = 0 \quad \text{or} \quad t = -1/8$$

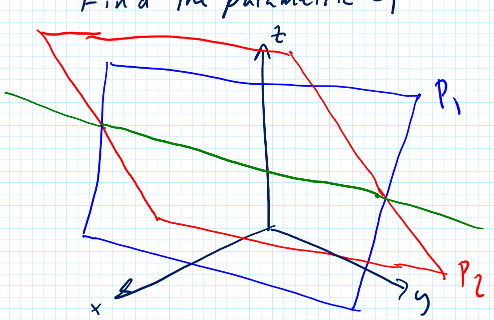
sub $t = -1/8$ into parametric eqn of line

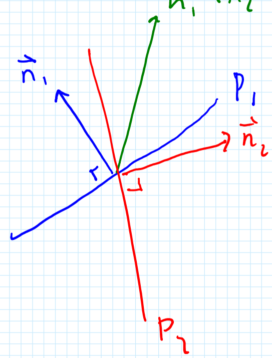
$$x = 1 - 2(-1/8) = 5/4$$
$$y = -1 + 3(-1/8) = -11/8$$
$$z = 3 - 4(-1/8) = 7/2$$

Ex. Consider the two planes $P_1$ given by $x + y - z + 4 = 0$
$P_2$ given by $2x - 3y + z - 3 = 0$

Find the parametric eqn of line of intersection

pt and direction




$\vec{n}_1 \perp \vec{n}_1 \times \vec{n}_2$

so $\vec{n}_1 \times \vec{n}_2$ parallel to $P_1$

$\vec{n}_2 \perp \vec{n}_1 \times \vec{n}_2$

so $\vec{n}_1 \times \vec{n}_2$ parallel to $P_2$

Punchline: $\vec{n}_1 \times \vec{n}_2$ is common parallel direction of intersection

$P_2$

Punchline : $\vec{n}_1 \times \vec{n}_2$ is common parallel direction = direction of intersection

$\vec{n}_1 = \langle 1, 1, -1 \rangle \qquad \vec{n}_2 = \langle 2, -3, 1 \rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ -3 & 1 \end{vmatrix} i - \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} j + \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} k$$

$$= -2i - 3j - 5k$$

$$= \langle -2, -3, -5 \rangle = -\langle 2, 3, 5 \rangle$$

Find a point in both $P_1$ and $P_2$.

$P_1 \quad x + y - z = -4$

$P_2 \quad 2x - 3y + z = 3$

Set $y = 0$

$$\begin{cases} x - z = -4 \\ 2x + z = 3 \end{cases}$$

$+$

$$3x + 0z = -1$$

$x = -1/3$

$y = 0$

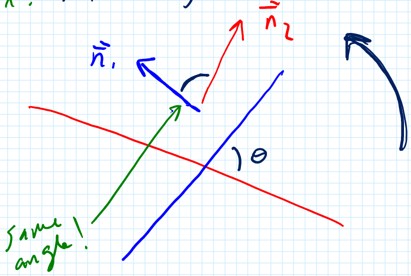
$z = 11/3$

Answer (parametric eqn of intersection)

$$\begin{cases} x = -1/3 + 2t \\ y = 0 + 3t \\ z = 11/3 + 5t \end{cases} \quad \text{or} \quad \begin{cases} x = -1/3 - 2t \\ y = 0 - 3t \\ z = 11/3 - 5t \end{cases}$$

$\langle 2,3,5 \rangle$ and $-\langle 2,3,5 \rangle$ determine same line

Ex. Find angle between $P_1$ and $P_2$




Rotate 90°

$$\cos(\theta) = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}$$

$\vec{n}_1 = \langle 1, 1, -1 \rangle, \ \vec{n}_2 = \langle 2, -3, 1 \rangle$

$|\vec{n}_1| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$

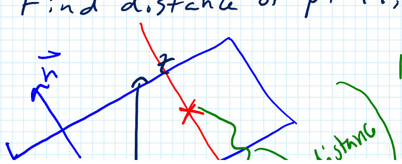
$|\vec{n}_2| = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$

$\vec{n}_1 \cdot \vec{n}_2 = 1 \cdot 2 + 1(-3) + (-1)1$

$= 2 - 3 - 1 = -2$

$$\cos\theta = \frac{-2}{\sqrt{3}\sqrt{14}}$$

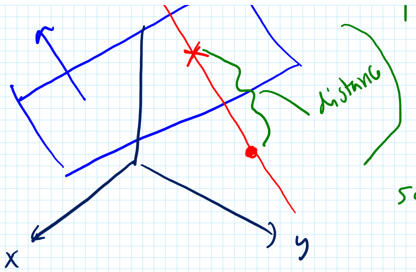
Ex. Find distance of pt $(1,2,0)$ to plane $x - y - 2z - 1 = 0$




line is $\langle 1, 2, 0 \rangle + t\vec{n}$

$x = 1 + t$

$y = 2 - t$




line is $(1,2,0) + t \ldots$

$x = 1 + t$
$y = 2 - t$
$z = \;\; -2t$

sub into plane

$(1+t) + (2-t) - 2(-2t) - 1 = 0$

$2 + 4t = 0, \quad t = -1/2$

Finds intersect of line w/ plane

pt $X \quad (\tfrac{1}{2}, 5/2, 1)$

Distance $(1,2,0)$ to plane is distance from $(1,2,0)$ to $(\tfrac{1}{2}, \tfrac{5}{2}, 1)$

$$= \sqrt{(1-1/2)^2 + (2-5/2)^2 + (0-1)^2}$$

$$= \sqrt{\tfrac{1}{4} + \tfrac{1}{4} + 1} = \sqrt{3/2}$$

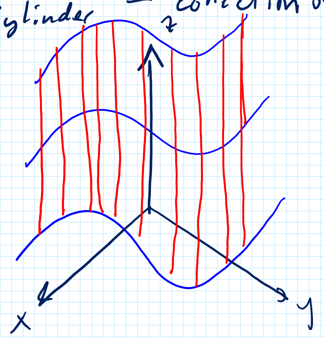
## § 12.6 Cylinders and Quadric Surfaces

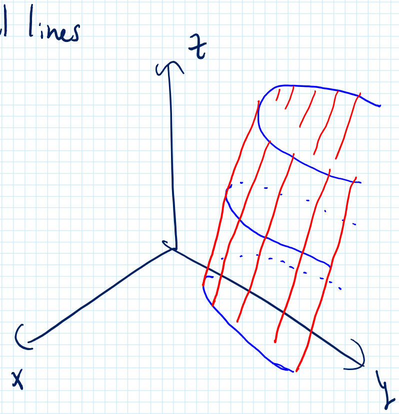
Up to rotation described by one of two equations

$$Ax^2 + By^2 + Iz = 0$$

$$Ax^2 + By^2 + Cz^2 + J = 0$$

Cylinder — collection of parallel lines

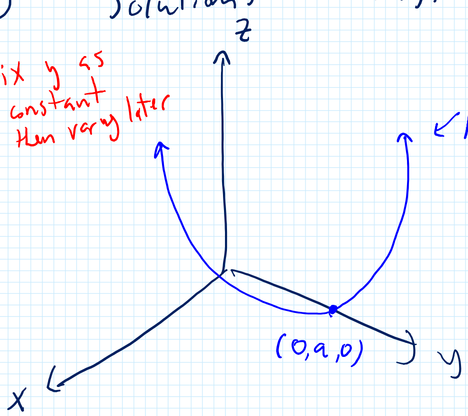


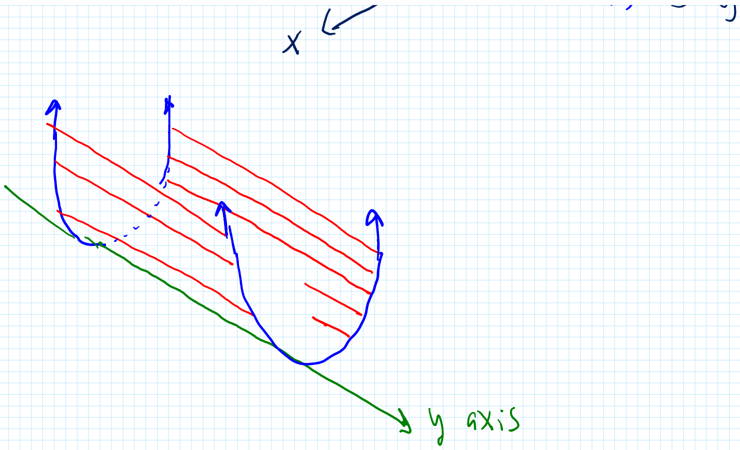




Ex. $x^2 - z = 0$ Solutions = all $(x,y,z)$ satisfying equ

$\begin{cases} y = a \\ z = x^2 \end{cases}$ fix y as constant then vary later

parabola $(x, a, x^2)$

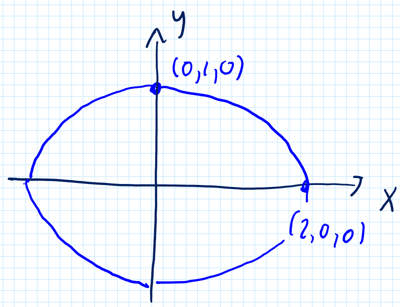


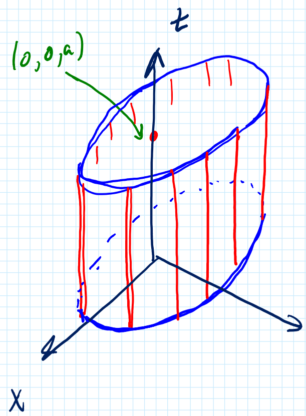



Ex. $x^2 + 4y^2 = 4$

$z = 0$

$$\begin{cases} \dfrac{x^2}{4} + y^2 = 1 \\ z = a \end{cases}$$




Elliptical cylinder

Ex. $x^2 + y^2 + 9z^2 = 9$

3 curves

3 curves

$z = 0 \quad x^2 + y^2 = 9$ ← circle radius 3

$y = 0 \quad x^2 + 9z^2 = 9$

ellipse

$x = 0 \quad y^2 + 9z^2 = 9$