261-34

Monday, November 12, 2018 1:01 PM

Math 261, Lecture 34, 11/12/18

Exam 2 returned in recitation tomorrow, 11/13

Today: §16.6 (begin), Next: §16.6 (finish)

Recap: $\vec{F}(x,y,z) = \langle P, Q, R \rangle$ a vector field

$$\vec{\nabla} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \text{ "del operator"}$$

$$\operatorname{curl} \vec{F} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$\operatorname{div} \vec{F} = \vec{\nabla} \cdot \vec{F} = P_x + Q_y + R_z$$

$$\operatorname{div}(\operatorname{curl} \vec{F}) = 0$$

In 2D $\vec{F} = \langle P, Q \rangle$ "$\operatorname{curl} \vec{F} \cdot \vec{k}$" $= Q_x - P_y$

$$\operatorname{div} \vec{F} = P_x + Q_y$$

Green's Thm
$$\oint_C \vec{F} \cdot \vec{T}\, ds = \oint_C P\,dx + Q\,dy = \iint_D \operatorname{curl} \vec{F} \cdot \vec{k}\, dA$$

$$\oint_C \vec{F} \cdot \vec{n}\, ds = \oint_C P\,dy - Q\,dx = \iint_D \operatorname{div} \vec{F}\, dA$$

F15

Problem 20: Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives up to second order and f a function with continuous partial derivatives up to second order. Which of the following is true?

F15 Final

tinuous partial derivatives up to second order and f a function with continuous partial derivatives up to second order. Which of the following is true?

✓ (1) $curl(grad(f)) = \mathbf{0}$ grad f is conservative so has curl = 0

✗ (2) $grad(div(\mathbf{F})) = \mathbf{0}$

✓ (3) $div(curl(\mathbf{F})) = 0$ known formula

✓ (4) $curl(curl(\mathbf{F}))$ is a vector field. curl of a vector field is still a vector field

✗ (5) $curl(div(\mathbf{F}))$ is a function.

div of $\vec{F}$ not a vector field

§16.6 Parametric Surfaces

Parametric curves C curve in 3D space

$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$\vec{r} = \begin{cases} x = t^3 - 3t + 1 \\ y = e^t \\ z = e^{-t} + t^2 \end{cases}$$

Parametrizing Surfaces

$$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$$

u, v parameters

$$\left.\begin{array}{l} a \le u \le b \\ c \le v \le d \end{array}\right\}$$ (u,v) are varying over rectangle $[a,b] \times [c,d]$

$$\left. u^2 + v^2 \le 9 \right\}$$ (u,v) varying over disk radius 3 centered at origin.

Ex. $$\left\{\begin{array}{l} x = u^2 - v^2 \\ y = u^2 + v^2 \end{array}\right\}$$ $y - x = 2v^2 = 2(-v)^2 = 2z^2$

Ex. $\begin{cases} x = u^2 - v^2 \\ y = u^2 + v^2 \\ z = -v \end{cases}$ $\Big\{ \; y - x = 2v^2 = 2(-v)^2$

surface lies on a parabolic cylinder

$$\begin{array}{rl} y = & u^2 + v^2 \\ - \quad x = & -u^2 + v^2 \\ \hline y - x = & 0 + 2v^2 \end{array}$$

Planes. P, plane need 3 vectors:

$r_0 = \langle x_0, y_0, z_0 \rangle$ in P

$\vec{a} = \langle a_1, a_2, a_3 \rangle$
$\vec{b} = \langle b_1, b_2, b_3 \rangle$ $\Big\}$ two distinct directions that are parallel to plane

$$\vec{r}(u,v) = \vec{r}_0 + \vec{a}\,u + \vec{b}\,v$$

or

$$\vec{r} = \begin{cases} x = x_0 + a_1 u + b_1 v \\ y = y_0 + a_2 u + b_2 v \\ z = z_0 + a_3 u + b_3 v \end{cases}$$

Ex. $\begin{array}{l} x = 1 + 5u - 7v \\ y = 0 + 3u + v \\ z = -2 + 1u - 2v \end{array}$

↳ $\langle 1, 0, -2 \rangle$ is in P $(u = 0 = v)$

$\vec{a} = \langle 5, 3, 1 \rangle$, $\vec{b} = \langle -7, 1, -2 \rangle$

Ex. $x + 2y + 3z = 12$, Parametrize

$$\left\{\begin{array}{l} x = u = 0 + 1u + 0v \\ y = v = 0 + 0u + 1v \\ z = 4 - \frac{1}{3}u - \frac{2}{3}v \end{array}\right. \qquad z = 4 - \frac{1}{3}x - \frac{2}{3}y$$

$$\vec{r}_0 = \langle 0, 0, 4 \rangle$$

$$\vec{a} = \langle 1, 0, -\tfrac{1}{3} \rangle$$

$$\vec{b} = \langle 0, 1, -\tfrac{2}{3} \rangle$$

Ex. parametrize $x^2 + y^2 + z^2 = 4$, $z \geq 0$

sphere radius 2

One way: solve for z, $z = \sqrt{4 - x^2 - y^2}$

standard coords

$$\left\{\begin{array}{l} x = u \\ y = v \\ z = \sqrt{4 - u^2 - v^2} \end{array}\right.$$

or

cylindrical coords

$$\left\{\begin{array}{l} x = u\cos(v) \\ y = u\sin(v) \\ z = \sqrt{4 - u^2} \end{array}\right. \qquad \begin{array}{l} 0 \leq u \leq 2 \\ 0 \leq v \leq 2\pi \end{array}$$

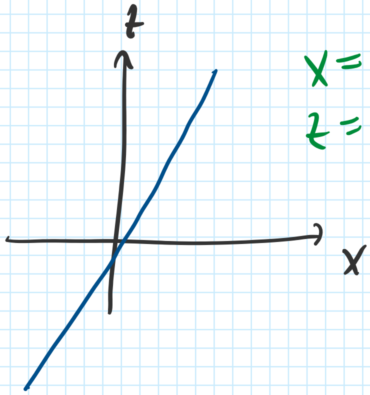
or

spherical coords

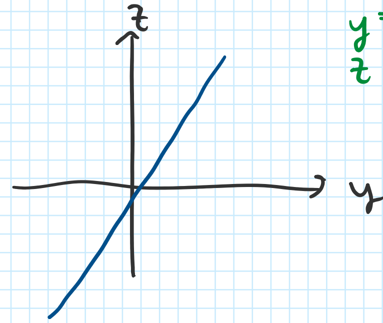
$$\left\{\begin{array}{l} x = 2\cos(u)\sin(v) \\ y = 2\sin(u)\sin(v) \\ z = 2\cos(v) \end{array}\right. \qquad \begin{array}{l} 0 \leq u \leq 2\pi \\ 0 \leq v \leq \pi/2 \end{array}$$

Ex. $\begin{cases} x = 2u \cos(v) \\ y = 3u \sin(v) \\ z = 6u \end{cases}$ Find the surface

angle of rotation. Surface of revolution around z-axis.



$\left.\begin{array}{l} x = 2u \\ z = 6u \end{array}\right\} \to z = 3x$



$\left.\begin{array}{l} y = 3u \\ z = 6u \end{array}\right\} \rightsquigarrow z = 2y$

We have a cone with elliptical profile, so given by

$$Ax^2 + By^2 = Cz^2 \qquad 1z^2$$

$$\|$$

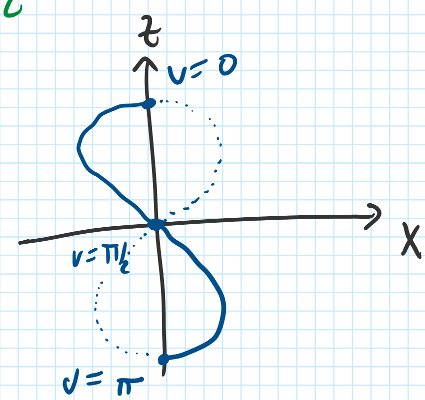
$$A(2u\cos(v))^2 + B(3u(\sin v))^2 = 36u^2$$

$$4A\, u^2 \cos^2 v + 9B\, u^2 \sin^2 v = 36u^2$$

$A = 9$ and $B = 4$, $C = 1$

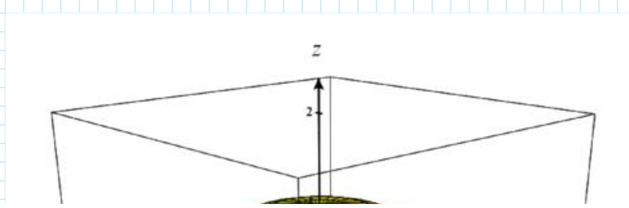
Ex. $\begin{cases} x = \cos(u)\sin(2v) \\ y = \sin(u)\sin(2v) \\ z = \cos(v) \qquad 0 \le u, v \le 2\pi \end{cases}$

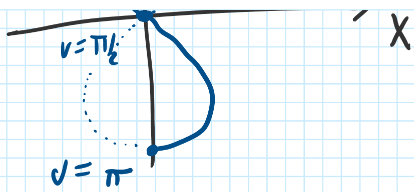
$x = \cos(u)\ldots$
$y = \sin(u)\ldots$
so rotates around z-axis.

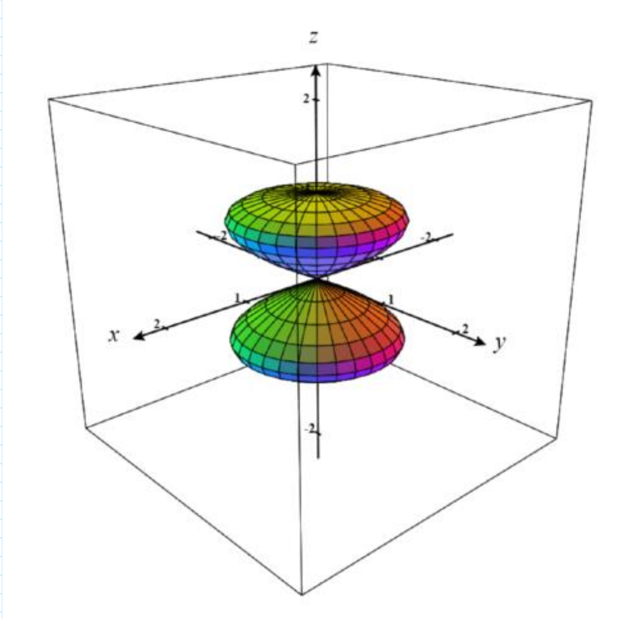


$u = 0$ on xz plane

so $x = \sin(2v)$
$z = \cos(v)$




$v = \pi/2$
$v = \pi$
X


z
x
y