Math 261, Lecture 4, 8/27/18

Today: §12.6 Quadric Surfaces, finish ↳ Next §13.1

Announcements
- No need to repurchase WebAssign lifetime access ↳ Cengage Unlimited ≠ WebAssign
- WebAssign
  ↳ "My Cengage Home" for status
- WebAssign comes with e-book

§12.6, Quadric Surfaces (cont'd)

Recap: linear (degree 1) surface = Plane (§12.5)

$$Ax + By + Cz + D = 0$$

quadric (degree 2) surface → There are many (6) types

$$\left.\begin{aligned} &Ax^2 + By^2 + Iz = 0 \\ &Ax^2 + By^2 + Cz^2 + J = 0 \end{aligned}\right\} \text{Two basic eqns up to symmetries}$$

↳ $A, B,$ or $C = 0$ cylinder

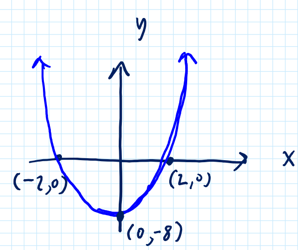
Sketching Quadric Surfaces | "Method of Traces"

In 2D there are 3 types of quadratics: (See §10.5 Conic Sections)

1) $ax^2 + by + c = 0$ (parabola)

↳ ex. $2x^2 - y - 8 = 0$

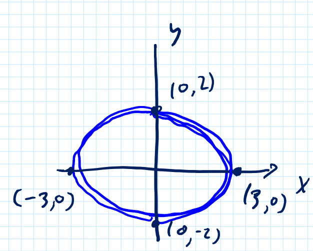
$y = 2x^2 - 8$



2) $ax^2 + by^2 = c$

↳ $a, b, c > 0$ (ellipse)

↳ ex. $\frac{x^2}{9} + \frac{y^2}{4} = 1$



3) $ax^2 + by^2 = c$

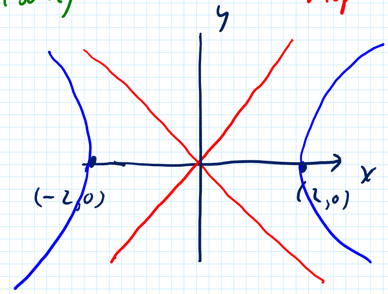
asymptotes

3) $ax^2 + by^2 = c$

$\hookrightarrow ab < 0$ (hyperbola)

$\hookrightarrow \frac{x^2}{4} - \frac{y^2}{9} = 1$

asymptotes slope $= \pm 3/2$



$x = 0$
$-\frac{y^2}{9} = 1$
No sol'ns

$y = 0$ $\frac{x^2}{4} = 1$
$x^2 = 4$
$x = \pm 2$

Method of Traces − Fix one coordinate x, y, z and find corresponding 2D conic.

$\hookrightarrow$ See Table 1 p. 837

Ex. $\frac{x^2}{4} + y^2 - z^2 = 0$

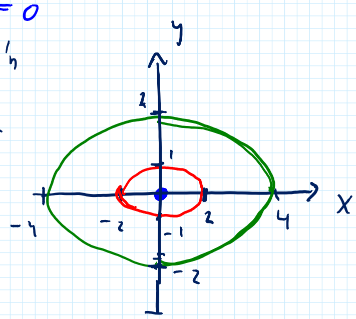
Horizontal traces $z = k$

$k = 0$ (xy axis) $\frac{x^2}{4} + y^2 = 0 \rightarrow x = y = 0$
$\hookrightarrow$ single pt sol'n

$k = 1$ $\frac{x^2}{4} + y^2 = 1 \rightarrow$ ellipse

$k = 2$ $\frac{x^2}{4} + y^2 = 4 \rightarrow$ ellipse larger radius

? $k = -1$ same as $k = 1$



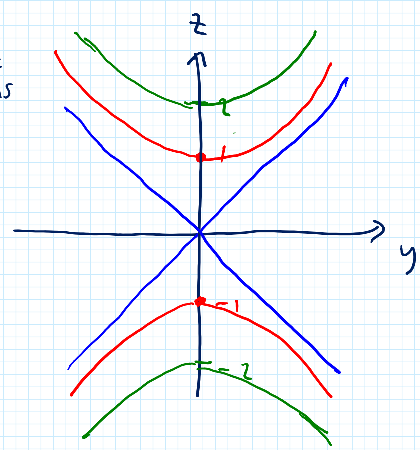
Vertical traces $\hookrightarrow x = k$

$k = 0$ $y^2 - z^2 = 0 \longrightarrow y = \pm z$

$k = 2$ $y^2 - z^2 = -1 \longrightarrow$ hyperbola
y and z have different signs

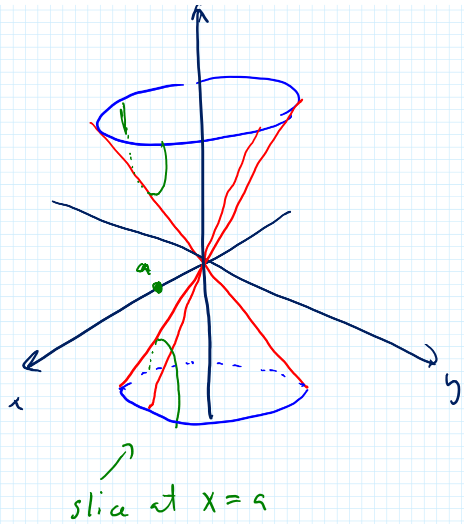
$k = 4$ $y^2 - z^2 = -4 \longrightarrow$

? $\left.\begin{array}{l} k = -2 \\ k = -4 \end{array}\right)$ ± symmetry on X-axis



Vertical traces
$\hookrightarrow y = k$, Similar picture

All together



Ex. $x^2 + y^2 - z^2 + 1 = 0$

Horizontal traces, $z = k$

$k = 0$ $\quad x^2 + y^2 = -1$

↳ no sol'ns $\quad -1 < z < 1$

$k = 1$ $\quad x^2 + y^2 = 0$

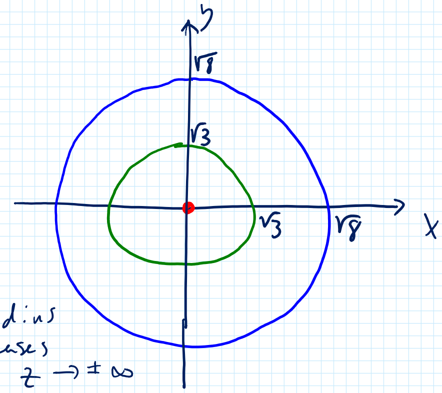
pt $(0,0)$

$k = 2$ $\quad x^2 + y^2 = 3$

$k = 3$ $\quad x^2 + y^2 = 8$

radius increases $z \to \pm\infty$

$k = -1$
$k = -2$) symmetry $\pm k$



Vertical Traces

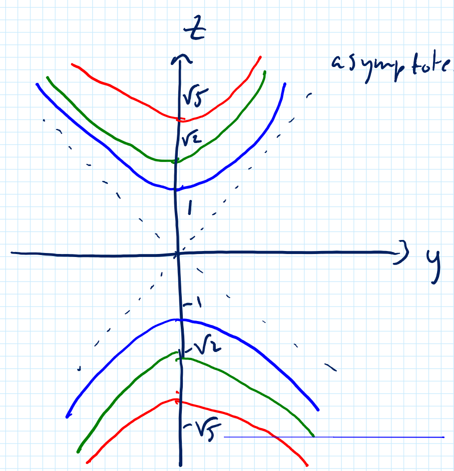
↳ $x = k$

$k = 0$ $\quad y^2 - z^2 = -1$

$k = 1$ $\quad y^2 - z^2 = -2$

$k = 2$ $\quad y^2 - z^2 = -5$

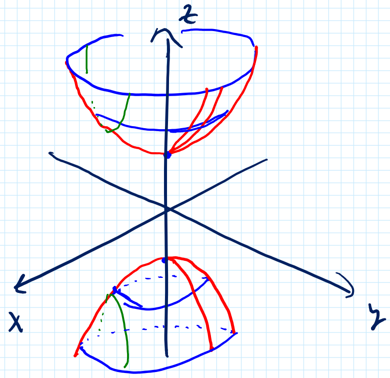
asymptotes $y = \pm z$



Vertical Traces

$\hookrightarrow$ $y = k$ Similar to $x = k$

All together,



z term is linear

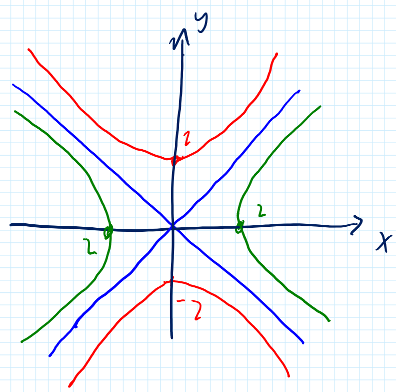
Ex. $-x^2 + y^2 - 4z = 0$

Horizontal traces, $z = k$

$k = 0 \quad -x^2 + y^2 = 0 \quad y = \pm x$

$k = 1 \quad -x^2 + y^2 = 4$

$k = -1 \quad -x^2 + y^2 = -4$



Vertical Traces $\hookrightarrow$ $x = k$

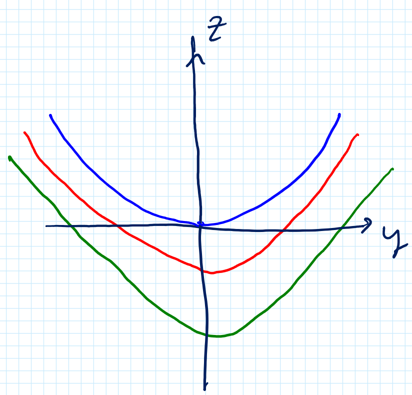
$k = 0 \quad y^2 - 4z = 0$

$k = 1 \quad y^2 - 4z = 1$

$k = -1 \quad y^2 - 4z = 1$

$k = 2 \quad y^2 - 4z = 4$

$z = \frac{y^2}{4} - 1$

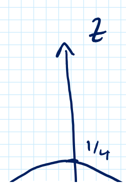


Vertical Traces $\hookrightarrow$ $y = k$

$-x^2 + y^2 - 4z = 0$

$k = 0 \quad -x^2 - 4z = 0$

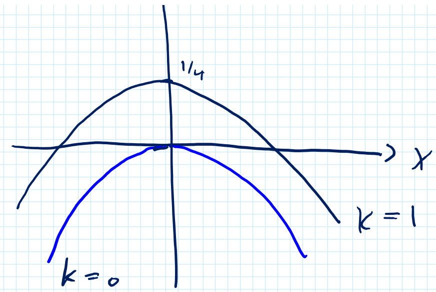
$z = -\frac{x^2}{4}$



$$z = -\frac{x^2}{4}$$

$k = 1 \qquad -x^2 - 4z = -1$

$$z = \frac{x^2}{4} + \frac{1}{4}$$



All together

Figure 8, p. 836

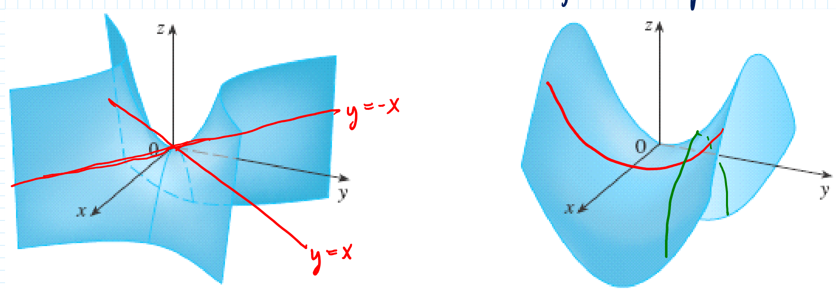


Table 1 p. 837

Graphs of quadric surfaces

Surface	Equation	Surface	Equation
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid. linear z	Hyperboloid of One Sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

Be able to identify surface based on equation / traces

Ex. Write the quadric $2x^2 - 6y^2 + 2x + 12y - z = 7$ in standard form.

Complete the square

$$2x^2 + 2x - 6y^2 + 12y - z = 7$$

$$2\left(x^2 + x + \tfrac{1}{4}\right) - 6\left(y^2 + 2y + 1\right) - z = 7 + 2\cdot\tfrac{1}{4} - 6\cdot 1$$

$$\boxed{2\left(x + \tfrac{1}{2}\right)^2 - 6\left(y+1\right)^2 - z = \tfrac{3}{2}}$$

hyperbolic paraboloid

Ex.* Write $3x^2 + xy + 3y^2 - 7z^2 = 3$ in standard form

Ex. $x^2 + y^2 - z^2 - 1 = 0$