Math 261, Lecture 40, 12/3/18

Today: Review, Next: Review

- FINAL EXAM, WED 12/12, 8AM
  ELLIOTT HALL
  - Conflicts!
  - > 2 exams in one day, can reschedule any one
- Office Hours (MATH 744)
  M, 12/3, 3:30-5:00
  W, 12/5, 4:30-6:30
  F, 12/7, 11:30-12:30, 3:30-5:00
  M, 12/10, 2:30-4:30
  T, 12/11, 3:30-5:30

  or by appointment

Recap: 4 Fundamental Identities

- Fundamental Thm of Line Integrals

$$\int_C \vec{\nabla} f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

- Green's Thm

$$\oint_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$$

Use these for a surface w/ boundary

sphere of radius 2 above $z=1$

- Stokes's Thm

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_S \operatorname{curl}\vec{F}\cdot d\vec{S}$$

$\partial S = C$, positive orientations on both

upward = outward

- Divergence Thm

$$\underset{\text{outward}}{\iint_S} \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}(\vec{F})\,dV$$

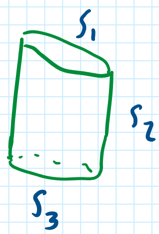
no dot

$$\partial E = S$$

Ex. complicated <u>closed</u>, surface

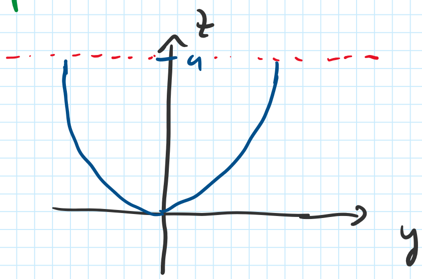
– regions bounded by cylinders and planes

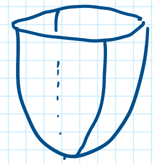
– Compute $\iint_S \vec{F}\cdot d\vec{S}$ for:




– hemisphere w/ bottom

Ex. (518 #19) $\quad z = x^2 + y^2$



single surface
w/ boundary

Compute $\displaystyle\iint_S \text{curl}(\vec{F})\cdot\vec{n}\, dS$

$$dS = \vec{r}_u \times \vec{r}_v \, dA$$
$$= \vec{n}\, dA \quad (\vec{n}\, dS)$$

$$\iint_S \text{curl}(\vec{F})\cdot d\vec{S}$$

flux = vector field • normal

curl, so think Stokes

$$= \int_{\partial S} \vec{F}\cdot d\vec{r}$$

$\partial S \qquad 4 = z = x^2 + y^2$

circle radius 2 in the plane $z = 4$.

parametrize $\partial S$ $\qquad \vec{r}(t) = \begin{cases} x = 2\cos(t) \\ y = 2\sin(t) & 0 \le t \le 2\pi \\ z = 4 \end{cases}$

$$\vec{F} = 0i + xz\, j + yz\, k$$

$$\oint_{\partial S} \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\, dt$$

$$\oint_{\partial S} \vec{F} \cdot d\vec{r} = \int_0 \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

$$\vec{F}(\vec{r}(t)) = \langle 0, 8\cos(t), 8\sin(t) \rangle$$

$$\vec{r}\,'(t) = \langle -2\sin(t), 2\cos(t), 0 \rangle$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t) = 0 + 16\cos^2(t) + 0 = 16\cos^2(t)$$

$$= \int_0^{2\pi} 16\cos^2(t) = 16\pi$$

Ex. (F17, #12) Find $\int_C \vec{F} \cdot d\vec{r}$

Try to write $\vec{F}$ as $\vec{\nabla} f$

$$\vec{F}(x,y,z) = ye^x\, i + e^x\, j + 2z\, k$$

$$ye^x + h(x,z) \xrightarrow{\frac{\partial}{\partial z}} 2z$$

$$h_z = 2z$$

$$h = z^2 + g(x)$$

$$ye^x + z^2 + g(x)$$

$$\frac{\partial}{\partial x}: \quad ye^x + \underbrace{g'(x)}_{0}$$

$$\vec{F} = \vec{\nabla}(\underbrace{ye^x + z^2}_{f})$$

By Fund Thm of Line Integrals

By Fund Thm of Line Integrals

$$\int_C \vec{F} \cdot d\vec{r} = f(0,1,0) - f(0,0,2)$$

$$= 3 - 4 = -1$$

Ex. (F17, #14) $\vec{F} = x^2yz\, i + xy^2z\, j + xyz^2\, k$

grad(div $\vec{F}$) − curl(curl $\vec{F}$)

$$\operatorname{div} \vec{F} = P_x + Q_y + R_z = 2xyz + 2xyz + 2xyz$$

$$= 6xyz = f$$

$$\operatorname{grad} f = \langle f_x, f_y, f_z \rangle$$

$$= \langle 6yz, 6xz, 6xy \rangle$$

$$\operatorname{curl} \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2yz & xy^2z & xyz^2 \end{vmatrix}$$

$i$

$$= (xz^2 - xy^2)\, i + (-yz^2 + x^2y)\, j + (y^2z - x^2z)\, k$$

$$\operatorname{curl}(\operatorname{curl}(\vec{F})) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^2 - xy^2 & x^2y - yz^2 & y^2z - x^2z \end{vmatrix}$$

$$= (2yz + 2yz)i + (2xz + 2xz)j + (2xy + 2xy)k$$

$$= \langle 4yz, 4xz, 4xy \rangle$$

So $\operatorname{grad}(\operatorname{div} \vec{F}) - \operatorname{curl}(\operatorname{curl} \vec{F})$

$$= \langle 6yz, 6xz, 6xy \rangle - \langle 4yz, 4xz, 4xy \rangle$$

$$= \langle 2yz, 2xz, 2xy \rangle$$