261-5

Wednesday, August 29, 2018 11:31 AM

Math 261, Lecture 5, 8/29/18

Outline §13.1 all, §13.2 begin (maybe)
↳ Next §13.2 (all)

Recap: §12.6 Quadric Surfaces

Use Method of Traces

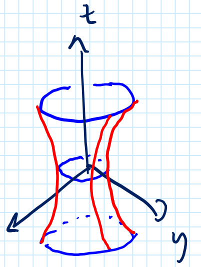
Ex. $x^2+y^2-z^2-1=0$

$z=0$ $x^2+y^2=1$ (ellipse xy plane)

$z=k$ always have ellipses

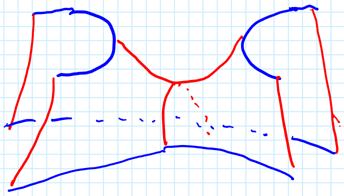
$x=0$ $y^2-z^2=1$ — hyperbolas

$y=0$ $x^2-z^2=1$



hyperboloid of one sheet

Ex.



hyperbolic paraboloid

$$\underbrace{x^2-y^2}_{\text{hyperbolic}} - \underset{\nwarrow\,\text{paraboloid}}{z} - 1 = 0$$

§13.1 Curves in 3D

$$\vec{r}(t) = \langle \underset{\uparrow}{f(t)}, \underset{\nearrow}{g(t)}, \underset{\nearrow}{h(t)} \rangle$$

components

$$\vec{r}(t) = f(t)\, i + g(t)\, j + h(t)\, k$$
$$= f(t)\langle 1,0,0\rangle + g(t)\langle 0,1,0\rangle + h(t)\langle 0,0,1\rangle$$

$+ h(t)\langle 0,0,1\rangle$

$$= f(t)\langle 1,0,0\rangle + g(t)\langle 0,1,0\rangle + h(t)\langle 0,0,1\rangle$$

parametric

$x = f(t)$
$y = g(t)$
$z = h(t)$

Ex. $\vec{r}(t) = \langle e^{t}, \sin(t), t\sinh(t)\rangle$

$\vec{r}(t) = e^{t} i + \sin(t) j + t\sinh(t) k$

$x = x(t) = e^{t}$
$y = y(t) = \sin(t)$
$z = z(t) = t\sinh(t)$

$$\boxed{\begin{aligned}\cosh(t) &= \frac{e^{t}+e^{-t}}{2}\\ \sinh(t) &= \frac{e^{t}-e^{-t}}{2}\end{aligned}}$$

⟵ See §3.11 p. 259 for hyperbolic trig fxns

Limits and Continuity

$\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$

$$\lim_{t\to a} \vec{r}(t) = \left\langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\right\rangle$$

Ex. $\vec{r}(t) = \left\langle t\ln(t), t, \frac{\sin(t)}{t}\right\rangle$

$$\lim_{t\to 0} \vec{r}(t) = \left\langle \underbrace{\lim_{t\to 0} \cancel{t}\ln(t)}_{0}, \underbrace{\cancel{\lim_{t\to 0} t}}_{0}, \underbrace{\lim_{t\to 0} \cancel{\frac{\sin(t)}{t}}}_{1}\right\rangle$$

$$\boxed{\lim_{t\to 0} t\ln(t) = \lim_{t\to 0} \frac{\ln(t)}{\frac{1}{t}} = \lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t\to 0} -t}$$

L'Hospital's Rule

Answer $\langle 0,0,1\rangle$

Continuity is $\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$

Same as all components are continuous

Sketching Curves

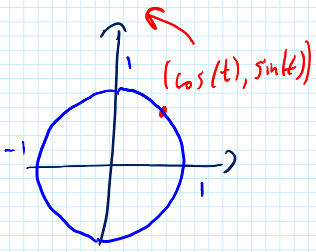
Ex. $\vec{r}(t) = \cos(t) i + \sin(t) j + t k$

$x = \cos(t)$
$y = \sin(t)$
$z = t$

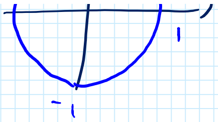
$\uparrow t$

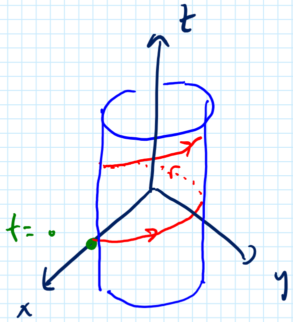
xy plane

$t = 0$ $\vec{r}(0) = \langle 1,0,0\rangle$



$t = 0 \quad \vec{r}(0) = \langle 1, 0, 0 \rangle$





Ex. $\vec{r}(t) = \langle \sqrt{t}\cos(t), \sqrt{t}\sin(t), 0.3t \rangle$

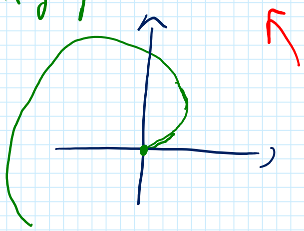
$x = \sqrt{t}\cos(t)$

$y = \sqrt{t}\sin(t)$

$z = 0.3t$

$t \geq 0$

xy plane



$x^2 + y^2 = t\cos(t)^2 + t\sin(t)^2$

$= t(\cos(t)^2 + \sin(t)^2)$

$= t \cdot 1 = t$

What surface is the curve on?

$x^2 + y^2 = t = \frac{10}{3}z$

$\hookrightarrow \underbrace{x^2 + y^2}_{\text{elliptic}} - \frac{10}{3}z = 0$ for values of $t \geq 0$

↑ paraboloid

§13.2 Derivatives and Integrals

$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

$$\frac{d}{dt}\vec{r}(t) = \lim_{h \to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

$\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$

Bonus Examples! for §13.1

Basic identity

$t\cosh(t)$

Ex. $x = t\cosh(t)$

$y = 3t\sinh(t)$

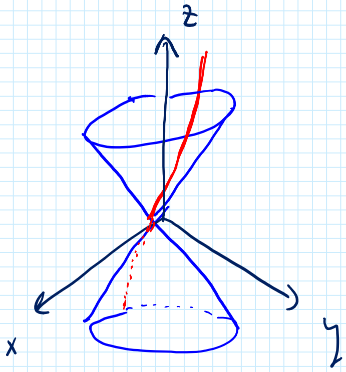
$z = 2t$

Basic identity

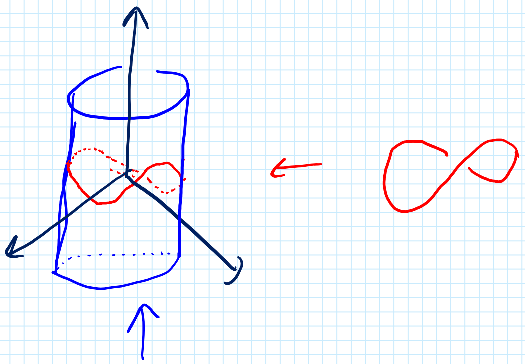
$\cosh(t)^2 - \sinh(t)^2 = 1$

$$\begin{aligned} 9x^2 + y^2 &= 9(t\cosh(t))^2 - (3t\sinh(t))^2 \\ &= 9t^2\cosh(t)^2 - 9t^2\sinh(t)^2 \\ &= 9t^2\left[\cosh(t)^2 - \sinh(t)^2\right] \\ &= 9t^2 \\ &= \frac{9}{4}z^2 \end{aligned}$$

So curve moves on surface $\underbrace{9x^2 + y^2 - \frac{9}{4}z^2 = 0}_{\text{cone}}$



Ex. Curve that is intersection of $x^2+y^2=1$ and $2x^2-3y^2-z=1$



$x^2+y^2=1$ cylinder

$x = \cos(t)$
$y = \sin(t)$ $\quad > \quad x^2 + y^2 = \cos(t)^2 + \sin(t)^2 = 1$

for all t

Plug in to equation of other surface

$$z = 1 - 2x^2 + 3y^2$$

$$= 1 - 2(\cos(t))^2 + 3(\sin(t))^2$$

$$= 1 - 2\cos(t)^2 + 3\sin(t)^2$$

$$= 1 + 2(1 - \sin(t)^2) + 3\sin(t)^2$$

$$= 3 + 5\sin(t)^2$$

Answer parametric form of curve of intersection

$x = \cos(t)$
$y = \sin(t)$
$z = 3 + \sin(t)^2$