Math 261, Lecture 9, 9/10/18

Today: §14.1, Next: §14.2

Recap. §13.4, Velocity and Acceleration

$\vec{r}(t) = f(t)\,i + g(t)\,j + h(t)\,k$

$\vec{v}(t) = \vec{r}\,'(t)$ velocity vector $|\vec{v}(t)|$ = "speed"

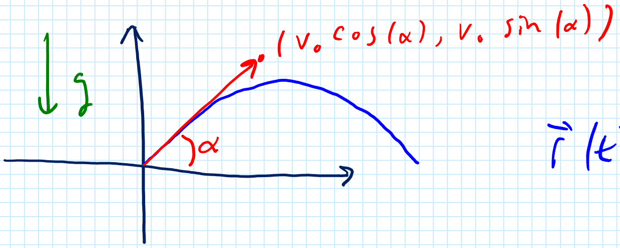
$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$ acceleration vector

$\rightarrow$ FTC $\vec{v}(t) = \langle f(t), g(t), h(t) \rangle$

then $\vec{r}(t) = \langle \int f(t)\,dt + C_1, \int g(t)\,dt + C_2, \int h(t) + C_3 \rangle$

$= \langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \rangle + \langle C_1, C_2, C_3 \rangle$

$= \int \vec{v}(t)\,dt + \vec{C}$



$\vec{r}(t) = \left\langle v_0 \cos(\alpha)\,t,\ v_0 \sin(\alpha)\,t - \frac{g}{2}t^2 \right\rangle$

Chapter 14: Partial Derivatives

§14.1, Functions of Several Variables

Up to now: fxns of a single variable

- $\vec{W}(t) = \langle$ temp, humidity, wind speed $\rangle$

  t time as measured at Purdue airport

functions of two $\{$ • h(humidity, temp) = heat index

functions of two variables
- $h(\text{humidity}, \text{temp}) = \text{heat index}$
- $T(x,y)$ temp at GPS coord (x,y)

$z = f(x,y)$ fxn of two variables, x and y.

<u>Domain</u> all (x,y) in the plane where $f(x,y)$ is defined.

Ex. $f(x,y) = \ln(-x^2+y^2-9)$

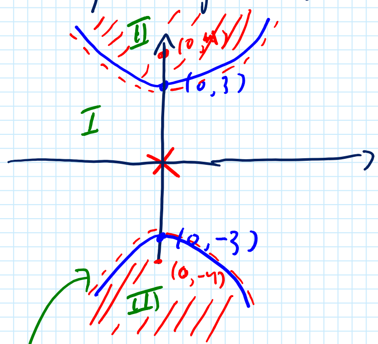
Find domain — last operation first and work inwards

Last operation $\ln(\ >0\ )$

<u>Need.</u> $-x^2+y^2-9>0$

Set equality to get boundary $-x^2+y^2=9$

hyperbola is the boundary curve



boundary curve is <u>not</u> in domain

Test the inequality at a point in each region

$(0,0)$ $\quad -(0)^2+(0)^2-9 \overset{?}{>} 0$ No!

$(0,4)$ $\quad -(0)^2+(4)^2-9 \overset{?}{>} 0$ Yes!

$(0,-4)$ $\quad -(0)^2+(-4)^2-9 \overset{?}{>} 0$ Yes!

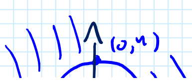
Ex. $f(x,y) = \sqrt{x^2+y^2-16} + \dfrac{1}{\sqrt{9-x^2}}$

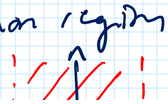
Find the domain

$\sqrt{\geq 0}$ $\qquad \dfrac{1}{(\neq 0)}, \ \dfrac{1}{\sqrt{\geq 0}}$

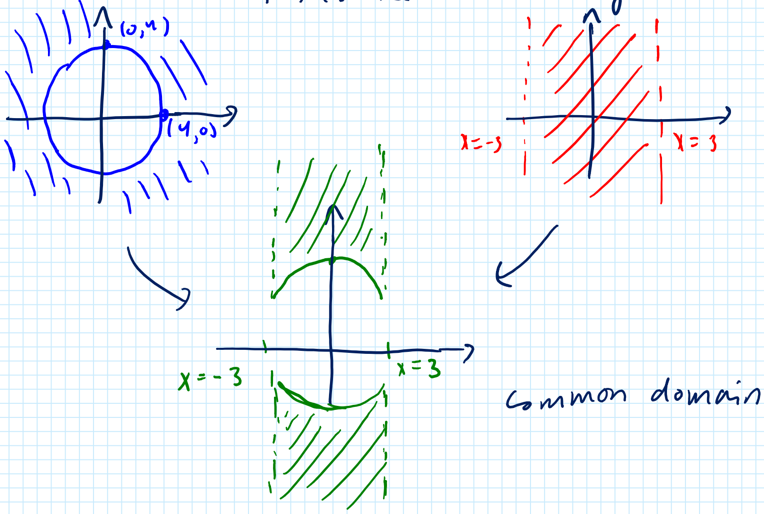
$x^2+y^2-16 \geq 0$ $\qquad 9-x^2>0$ <u>or</u> $-3<x<3$

Find the common region





Find the common region

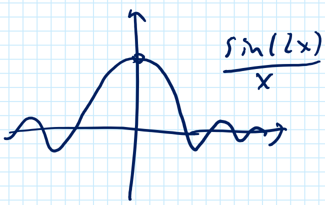


Ex. $f(x,y) = \dfrac{\sin(2x)\sin(2y)}{xy}$

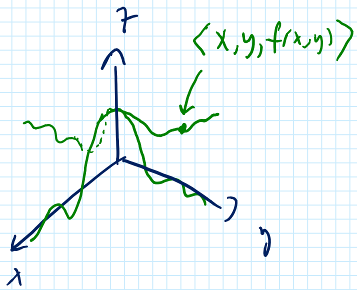
$x=0$ or $y=0$ Not in domain

We can "patch" this function up to the plane as we will see in 14.2

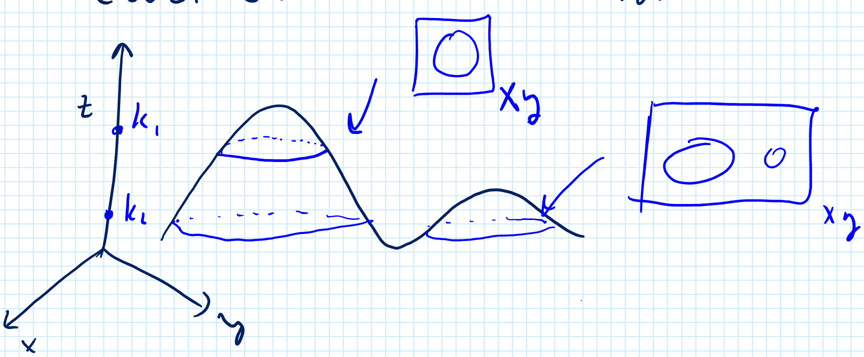
Fix $y \neq 0$ $x \to 0$ $\dfrac{\sin(2x)}{x}\left[\dfrac{\sin(2y)}{y}\right]$ x-limit exists!



$z = f(x,y)$



Level curves $z = f(x,y)$



Ex. $z = \dfrac{10x}{1+10x^2+10y^2}$

Find level curves

Fix $z=k$ $\quad k = \dfrac{10x}{1+10x^2+10y^2}$ * See end of notes for full algebra details

$$10kx^2 + 10ky^2 - 10x = -k$$

↑ "x^2+y^2" pattern. Suggest level curves are circles. Find the centers

$$10k\left(x^2 - \frac{1}{k}x + \underline{\frac{1}{4k^2}}\right) + 10ky^2 = -k + \underline{10k\left(\frac{1}{4k^2}\right)}$$

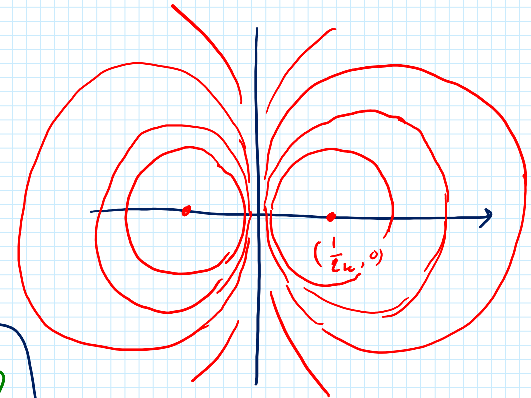
terms by completing the square

$$10k\left(x - \frac{1}{2k}\right)^2 + 10ky^2 = \frac{10}{4k} - k$$

$$\left(x - \frac{1}{2k}\right)^2 + y^2 = \frac{1}{4k^2} - \frac{1}{10}$$ ← This needs to be ≥ 0 so $\frac{1}{4k^2} \geq \frac{1}{10}$ or $-\frac{\sqrt{10}}{2} \le k \le \frac{\sqrt{10}}{2}$

circle centered at $\left(\frac{1}{2k}, 0\right)$

Since we divided by k we really need to use $\pm k$ so $\left(-\frac{1}{2k}, 0\right)$ are also centers



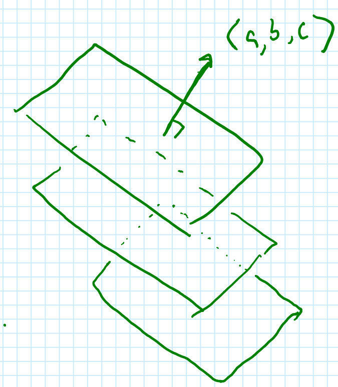
Level curves for $f(x,y) = \dfrac{10}{1+10x^2+10y^2}$

Ex. $w = f(x,y,z)$ $\quad f(x,y,z) = \vec{v}\cdot(x,y,z)$

$$\vec{v} = \langle a,b,c\rangle$$

Level surfaces?

$$k = ax + by + cz$$



level surfaces are parallel planes w/ normal $= \langle a,b,c\rangle$.

Derivation of Level Curves for $f(x,y) = \dfrac{10x}{1+10x^2+10y^2}$

$$k = \frac{10x}{1+10x^2+10y^2}$$

$$k(1+10x^2+10y^2) = 10x$$

$$k + 10kx^2 + 10ky^2 = 10x$$

$$10kx^2 - 10x \quad + 10ky^2 = -k$$

$$\hookrightarrow -10 = \frac{-10k}{k} = 10k\left(\frac{-1}{k}\right)$$

$$10k\left(x^2 - \frac{1}{k}x + \quad\right) + 10ky^2 = -k$$

$$\hookrightarrow x^2 - 2ax + a^2 = (x-a)^2$$

$-\frac{1}{k} = -2a$ or $a = \frac{1}{2k}$ so $a^2 = \frac{1}{4k^2}$

$$10k\left(x^2 - \frac{1}{k}x + \frac{1}{4k^2}\right) - 10ky^2 = -k + 10k\left(\frac{1}{4k^2}\right)$$

this gets multiplied into this and added to each side

$$10k\left(x - \frac{1}{2k}\right)^2 + 10ky^2 = -k + \frac{10}{4k}$$

$$10k\left[\left(x - \frac{1}{2k}\right)^2 + y^2\right] = -k + \frac{10}{4k}$$

$$\left(x^2 - \frac{1}{2k}\right)^2 + y^2 = \frac{-k}{10k} + \frac{10}{4k(10k)}$$