Notes on Hyperbolic Trig Functions.

$$\cosh(t) = \frac{e^{t} + e^{-t}}{2}$$

$$\sinh(t) = \frac{e^{t} - e^{-t}}{2}$$

Main Identity

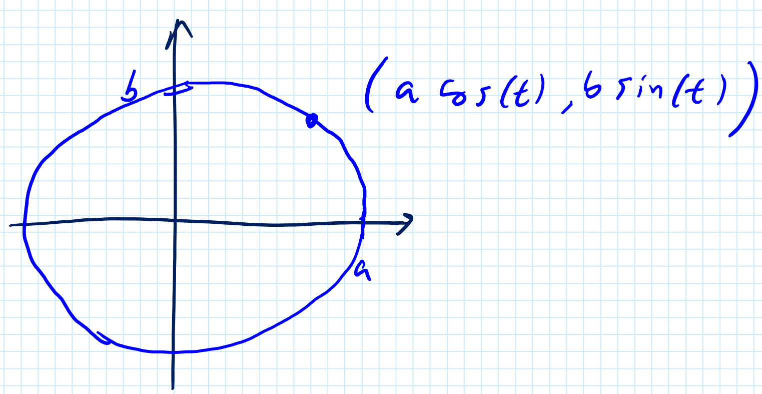
$$\boxed{\cosh(t)^2 - \sinh(t)^2 = 1}$$

Derivatives

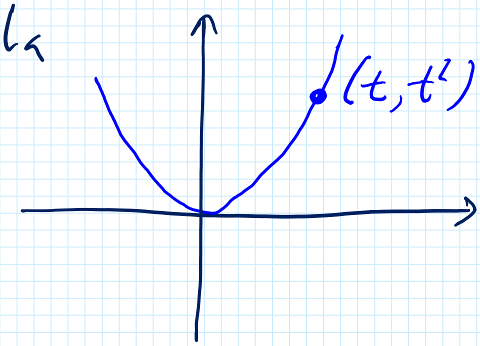
$$\frac{d}{dt}\cosh(t) = \sinh(t), \quad \frac{d}{dt}\sinh(t) = \cosh(t)$$

WTF! Well, they are really useful

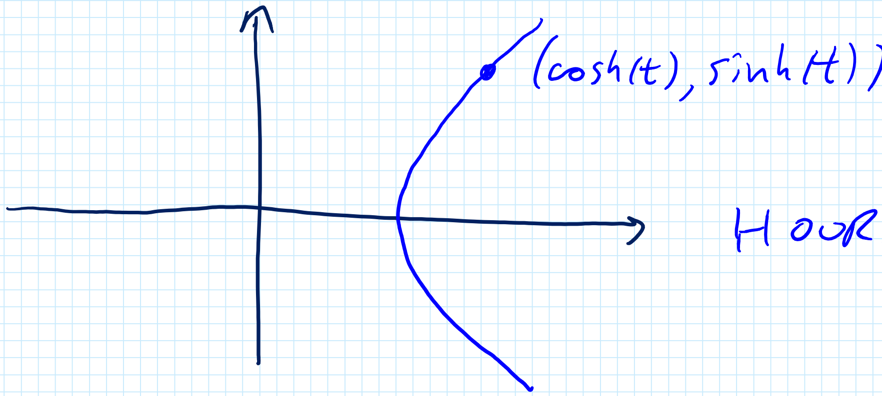
Recall ellipse



Recall parabola



Hyperbola



HOORAY!

Ex. Integrate $\displaystyle\int \frac{1}{\sqrt{1+x^2}}\,dx$

$$\left.\begin{aligned} \sinh(u) &= x \\ \cosh(u)\,du &= dx \end{aligned}\right\} \text{ hyperbolic trig substitution}$$

$$1 + \sinh(u)^2 = \cosh(u)^2$$

so $\displaystyle\int \frac{1}{\sqrt{1+x^2}}\,dx = \int \frac{\cosh(u)}{\sqrt{1+\sinh(u)^2}}\,du$

$$= \int \frac{\cosh(u)}{\cosh(u)}\,du = u + C$$

$$= \sinh^{-1}(x) = \ln\left(x+\sqrt{1+x^2}\right) + C$$

See Reference in back of book.

Ex. Compute $\displaystyle\int \sqrt{2+e^{-2x}+e^{2x}}\,dx$

$$\cosh(x)^2 = \left(\frac{e^{x}+e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}\left[(e^{x})^2 + e^{x}e^{-x} + e^{-x}e^{x} + (e^{-x})^2\right]$$

$$= \frac{1}{4}\left[e^{2x} + e^{(x-x)} + e^{(-x+x)} + e^{-2x}\right]$$

$$= \frac{1}{4}\left[e^{2x} + 2 + e^{-2x}\right]$$

Thus $2+e^{2x}+e^{-2x} = 4\cosh(x)^2$

so $\int \sqrt{2 + e^{-2x} + e^{2x}}\, dx = \int \sqrt{4\cosh(x)^2}\, dx$

$$= \int 2\cosh(x)\, dx$$

$$= 2\sinh(x) + C$$