Quiz 2

Find the maximal extent for a solution of the IVP

$$(50-t^2)y' + \sqrt{\frac{t-3}{t-5}}\,y = \frac{1-t}{\sin(t/2)} \qquad y(5.5) = -1$$

Solution. First convert to standard form for a 1st order linear eq'n:

$$y' + \underbrace{\frac{1}{50-t^2}\sqrt{\frac{t-3}{t-5}}}_{p(t)}\,y = \underbrace{\frac{1}{50-t^2}\cdot\frac{1-t}{\sin(t/2)}}_{g(t)}$$

Now find the largest common domain interval for $p(t)$ and $g(t)$ containing the initial t-value $t_0 = 5.5$

[Refer to blue box on page 69 for why this is true.]

$p(t)$ – domain $\quad t \neq \pm\sqrt{50} \sim \sqrt{49} = 7$

$\frac{t-3}{t-5} \geq 0 \rightsquigarrow \quad t \leq 3$ or $t > 5$

$g(t)$ domain $\quad t \neq \pm\sqrt{50}$

$\sin(t/2) \neq 0 \rightsquigarrow t \neq 0, 2\pi, -2\pi, 4\pi, \ldots$

5.5 in $(5, \sqrt{50})$ for $p(t)$ and $(0, 2\pi)$ for $g(t)$

So 5.5 in $(5, 2\pi)$ maximal extent of solution to IVP.