

Quiz 3

$$W(f, g) = 3e^{4t}, \quad f(t) = e^{2t}. \quad \text{Find } g(t).$$

$$\begin{aligned} W(f, g) &= \begin{vmatrix} f & f' \\ g & g' \end{vmatrix} = \begin{vmatrix} e^{2t} & 2e^{2t} \\ g & g' \end{vmatrix} \\ &= e^{2t}g' - 2e^{2t}g = 3e^{4t} \end{aligned}$$

divide by
 e^{2t}

$$\leadsto g' - 2g = 3e^{4t}/e^{2t} = 3e^{2t}$$

This is 1st order, linear, standard form so

integrating factor $\mu(t) = \exp(\int -2 dt) = e^{-2t}$

$$\leadsto \frac{d}{dt} [e^{-2t} g] = 3e^{2t} \cdot e^{-2t} = 3$$

$$\leadsto e^{-2t} g = \int 3 dt = 3t + C$$

$$\leadsto g(t) = 3te^{2t} + Ce^{2t}$$

(*) $g(t) = 3e^{2t}$ is also fine.

