Exam 4 Ground Rules

This exam is due Wednesday, December 14, by 9:00pm Eastern. No exceptions.

Treat this exam the same way as you would treat an in-class exam. In particular, this is a closed book, closed note, closed person, closed internet, etc. exam.

Do not read the exam until you are ready to complete it.

Please allow 1-2 hours of uninterrupted time in order to complete the exam.

The exam may be turned in in person, emailed as a pdf file, or placed in my mailbox in Math 419. The mailroom is closed after 5:00pm.

I will be available in my office until 4:30pm on Friday, December 9 and I will have regular office hours Monday, December 12.

I will be available in ME 1130, during the final examination time, 7:00-9:00pm on Wednesday, December 14.

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Exam 4

Work 4 problems for 5 points each.

Problem 0. Define the upper and lower Darboux integrals for a bounded function $f : [a, b] \to \mathbb{R}$. Define the Riemann integral of f if it exists.

Problem 1. Let $f : [a, b] \to \mathbb{R}$ be such that $f(x) \geq 0$ for all $x \in [a, b]$. Show f is integrable if for every n there is a step function $u(x) \geq f(x)$ such that $\int_a^b u(x)dx < 1/n$. Be sure to explain.

Problem 2. If $f : [a, b] \to \mathbb{R}$ is integrable, then $|f|$ is integrable as well. ($|f|$ is defined $|f|(x) = |f(x)|$.) Give an example that shows the converse is not true. Explain.

Problem 3. Let $f : [a, b] \to \mathbb{R}$ be differentiable with f' convex. Show that

$$\frac{f(b) - f(a)}{b - a} \leq (f'(b) + f'(a))/2.$$

Problem 4. For what values of p, q does

$$\lim_{t \to 0+} \int_t^1 \frac{\sin(x^p)}{x^q} dx$$

converge? Explain.

Problem 5. If $f : [1, \infty) \to \mathbb{R}$ is uniformly continuous and $\int_1^\infty f(t)dt < \infty$ show that $\lim_{x \to \infty} f(x) = 0$.