MATH 351, FALL 2017, HOMEWORK #4

DUE FRIDAY, SEPTEMBER 29

Problem 1 (Exercise 3.5 on Page 158)**.** Find a matrix A such that multiplication by A transforms the unit square $\{(0,0),(1,0),(0,1),(1,1)\}$ onto the parallelogram with corners $\{(0,0),(4,2),(2,4),(6,6)\}$.

Problem 2 (Exercise 3.7 on Page 158)**.** Find a 2×2 matrix B such that multiplication by A transforms the parallelogram with corners

$$\{(0,0),(2,1),(1,2),(3,3)\}$$

onto the parallelogram with corners

$$\{(0,0),(1,0),(1,1),(2,1)\}.$$

How many possible answers does this question admit?

Problem 3 (Exercise 3.8 on Page 158)**.** Show (using geometric reasoning) that the transformation which reflects points in $\mathbb{R}^2$ on the x axis is linear. Find a matrix that describes this transformation.

Problem 4 (Exercise 3.13(a)(b) on Page 160)**.** Although it seems silly, it is possible to do elementary row operations on $n \times 1$ matrices. Every such operation defines a transformation of $\mathbb{R}^3$ onto $\mathbb{R}^3$. For example, if we define a transformation of $\mathbb{R}^3$ into $\mathbb{R}^3$ by "add twice row 1 to row 3," this transformation transforms

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \to \begin{bmatrix} x_1 \\ x_2 \\ x_3 + 2x_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Since this transformation is described by a matrix, we see that our elementary row operation defines a linear transformation. A transformation defined by a single elementary row operation is called an **elementary transformation** and the matrix that describes such a transformation is called an **elementary matrix**.

Find matrices that describe the elementary row operations: (a) "Add twice row 3 to row 2" in $\mathbb{R}^4$ and (b) "Multiply row 2 by 17" in $\mathbb{R}^3$.

Problem 5 (Exercise 3.35 on Page 174)**.** Define a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ by the following rule: $T(X)$ is the result of first rotating X counterclockwise by $\frac{\pi}{6}$ radians and then multiplying by

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(a) What is the image of the unit square under T? (Rotation is always about the origin when it is considered as matrices unless otherwise specified)
(b) Find a matrix B such that $T(X) = BX$ for all $X \in \mathbb{R}^2$

Problem 6 (Exercise 3.36 on Page 175)**.** Suppose that in Exercise 3.35 (Problem 5 in this Homework) we multiply first by A and then rotate. Are your answers to parts (a) and (b) different? How?

Problem 7 (Exercise 3.44 on Page 175)**.** Find all 2×2 matrices B such that $AB = BA$ where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}.$$

Problem 8 (Exercise 3.49 on Page 176)**.** Find a 3×3 matrix A such that $A^3 = 0$ but $A^2 \neq 0$. [Hint: Try making most of the entries equal to zero.]