Quiz 1

Consider the family of vectors $\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ A \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \right\}$ in $\mathbb{R}^3$.

Find A so that this family is linearly independent, if such an A exists.

Solution. Linearly independent exactly when

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & A & 0 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right]$$ has a unique solution

Using Row Reduction

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & A+2 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & A+2 & 2 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 2A+6 & 0 \end{array}\right]$$

This has a unique solution exactly when each column contains a pivot, which means that $2A+6 \neq 0$ as the $2A+6$ entry must be the pivot for the third column.

Therefore, as long as $A \neq -3$ the system has a unique solution and ~~so~~ the vectors are ~~not~~ linearly independent.