Quiz 3

November 3, 2017

Find an orthogonal basis for $\mathbb{R}^4$ containing $\begin{bmatrix}1\\1\\1\\0\end{bmatrix}$.

Solution. $\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$ is clearly orthogonal to $\begin{bmatrix}1\\1\\1\\0\end{bmatrix}$

so we can either guess the next two vectors, or apply Gram-Schmidt to, for instance, the basis $\left\{\begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}\right\}$.

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} - \frac{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\\0\end{bmatrix}}{\|(1,1,1,0)\|^2}\begin{bmatrix}1\\1\\1\\0\end{bmatrix} - \frac{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{\|(0,0,0,1)\|^2}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\1\\1\\0\end{bmatrix} - 0\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}2/3\\-1/3\\-1/3\\0\end{bmatrix} = \frac{1}{3}\begin{bmatrix}2\\-1\\-1\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\1\\0\\0\end{bmatrix} - \frac{\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\\0\end{bmatrix}}{3}\begin{bmatrix}1\\1\\1\\0\end{bmatrix} - \frac{\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{1}\begin{bmatrix}0\\0\\0\\1\end{bmatrix} - \frac{\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\cdot\begin{bmatrix}2\\-1\\-1\\0\end{bmatrix}}{\|(2,-1,-1,0)\|^2}\begin{bmatrix}2\\-1\\-1\\0\end{bmatrix}$$

$$= \begin{bmatrix}0\\1\\0\\0\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\1\\1\\0\end{bmatrix} - 0\begin{bmatrix}0\\0\\0\\1\end{bmatrix} + \frac{1}{6}\begin{bmatrix}2\\-1\\-1\\0\end{bmatrix} = \begin{bmatrix}0\\1/2\\-1/2\\0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}$$

so $\left\{\begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}2\\-1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\0\end{bmatrix}\right\}$ is an orthogonal basis for $\mathbb{R}^4$ containing $\begin{bmatrix}1\\1\\1\\0\end{bmatrix}$.