

Quiz 4

Find the condition number of

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \|A\|_{1 \rightarrow 1} &= \max \left\{ \|(1, 0, 0, 0)^t\|_1, \|(-1, 1, 0, 0)^t\|_1, \|(0, -1, 1, 0)^t\|_1, \right. \\ &\quad \left. \|(0, 0, -1, 1)^t\|_1 \right\} \\ &= \max \{1, 2, 2, 2\} = 2 \end{aligned}$$

$$A = I + N \quad N = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad N^4 = 0$$

$$A^{-1} = I - N + N^2 - N^3$$

$$\begin{aligned} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \|A^{-1}\|_{1 \rightarrow 1} &= \max \left\{ \|(1, 0, 0, 0)^t\|_1, \|(1, 1, 0, 0)^t\|_1, \right. \\ &\quad \left. \|(1, 1, 1, 0)^t\|_1, \|(1, 1, 1, 1)^t\|_1 \right\} \\ &= \max \{1, 2, 3, 4\} = 4 \end{aligned}$$

$$\text{cond}(A) = \|A\|_{1 \rightarrow 1} \cdot \|A^{-1}\|_{1 \rightarrow 1} = 2 \cdot 4 = 8$$