•Modules over an arbitrary commutative ring. Kernel and image, direct sums and products, The isomorphism theorem and the fundamental theorem of linear algebra. Definition of exact sequences,

• Modules over principal ideal domains. The examples of a square matrix and f.g. abelian groups. Main theorem about f.g. modules over a PID. Application in the case of fixed linear operator over a finite dimensional vector space. Torsion factors and elementary factors of a linear operator and how to calculate these. Minimal polynomial. Rational canonical form and Jordan decomposition theorem. Eigenvalues, eigenvectors and characteristic polynomials. Cayley-Hamilton Theorem. Perron-Frobenius Theorem for Non-negative Matrices and Applications (including the Google search engine).

References: T.T.Moh Algebra, pp 196-214 or K.Hoffman & R Kunze pp 181-261.

• Several equivalent definitions of the determinant of a square matrix, including the axioms of determinants, and the Laplace formulas. The Cauchy-Binet formulas for the determinant. The multilinear produces and alternative produce of vector spaces and modules. The tensor product and exterior product of modules and vector spaces. Definitions and main properties of projective and free modules. Dual modules and double duals. Chain complex and cochain complex. Applications to advanced calculus. Definitions of exact sequences and resolutions. Describe Hilbert's Syzygies theorem and global dimension. Projective, injective and flat modules. Ext and Tor.

Reference: D. Dummit & R Foote Abstract Algebra pp339-381. 744-763

•Inner product spaces. Gram-Schmidt orthogonalization. Projections and the least square approximations. Definition of the adjoint of a ;inear operator. Definition and properties of self-adjoint and unitary operators. Definition of normal operators. The spectral theorem for normal operators over C.

• Linear Operators on inner product spaces, especially Hilbert space. Sesquilinear forms. Positive forms. Positive linear operators. The applications of positivities of the principal minors to the local minimal problem for n variables in calculus and high dimensional ellipsoids. Singular value decomposition theorem and its applications

• Bilinear forms. Symmetric bilinear forms. Matrix congrence. Sylvester's law of inertia. Signatures. Applications to physics and geometry. Lorantz group $SO(1, 3)$.

Reference K.Hoffman & R Kunze pp 270-385

There will be weekly homeworks, a mid-term and a final exam. The **main reference books** are K.Hoffman & R Kunze and my lecture notes which will be provided to students free.