When $k \geq 4$ and $0 \leq d \leq (k-2)/4$, we consider the system of Diophantine equations

$$x_1^j + \ldots + x_k^j = y_1^j + \ldots + y_k^j \quad (1 \leq j \leq k,\ j \neq k-d).$$

We show that in this cousin of a Vinogradov system, there is a paucity of non-diagonal positive integral solutions. Our quantitative estimates are particularly sharp when $d = o(k^{1/4})$.