My thesis work (advised by Peter Sarnak) shows that conditionally on some standard number theory conjectures (such as predictions of Random Matrix Theory type), almost all integers $n \not\equiv \pm 4 \mod 9$ are sums of three integer cubes. I will try to accurately convey the content of these conjectures, and discuss some of the ideas and themes behind my work (e.g. square-root cancellation, structure vs. randomness, and worst-case vs. average-case behavior).