Given a family of L-functions, there has been a great deal of interest in estimating the proportion of the family that does not vanish at special points on the critical line. Conjecturally, there is a symmetry type associated to each family which governs the distribution of low-lying zeros (zeros near the real axis). Generalizing a problem of Iwaniec, Luo, and Sarnak (2000), we address the problem of estimating the proportion of non-vanishing in a family of L-functions at a low-lying height on the critical line (measured by the analytic conductor). We solve the Fourier optimization problems that arise using the theory of reproducing kernel Hilbert spaces of entire functions (there is one such space associated to each symmetry type), and we can explicitly construct the associated reproducing kernels. We also address the problem of estimating the height of the "lowest" low-lying zero in a family for all symmetry types, a problem previously considered by Hughes and Rudnick (2003) and Bernard (2015). In this context, a new Fourier optimization problem emerges, and we solve it by establishing a connection to the theory of de Branges spaces of entire functions and using the explicit reproducing kernels we constructed. This is joint work with Emanuel Carneiro (ICTP) and Andrés Chirre (NTNU).