

The sum-product problem is a central one in arithmetic combinatorics. It's an uncertainty principle which concerns the interaction of addition and multiplication, and has different interpretations wherever those two operations make sense. If addition and multiplication are indeed incompatible, then it stands to reason that most non-trivial combinations of the two operations should result in something rather complicated. This is called an expansion result, and for a long time, these sorts of results were usually deduced (over the real numbers) using combinatorial geometry. The idea is that multiplication, relative to addition, is convex. It turns out that convexity is really only a first order approximation to the truth, and higher order convexity – a phenomenon undetected by traditional geometric methods – leads to better results. I'll discuss this and how it allows us to turn the tables, deducing geometric results from better expansion estimates.