SOME NEW PROGRESS ON IGUSA'S CONJECTURE FOR EXPONENTIAL SUMS

KIEN HUU NGUYEN

ABSTRACT. Let $f \in \mathbb{Z}[x_1, ..., x_n]$ be a non-constant polynomial. Let p be a prime number and m be a positive integer. We associate to f, p, m the exponential sum

$$E_f(p,m) := \frac{1}{p^{mn}} \sum_{x \in (\mathbb{Z}/p^m\mathbb{Z})^n} \exp(2\pi i f(x)/p^m).$$

Let σ be a positive real number. Suppose that for each prime number p, there is a positive constant c_p such that

$$|E_f(p,m)| \leq c_p p^{-m\sigma}$$

for all $m \geq 2$. Igusa's conjecture for exponential sums predicts that one can take c_p independent of p in the above inequality. This conjecture relates to the existence of a certain adèlic Poisson summation formula and the estimation of the major arcs in the Hardy-Littlewood circle method towards the Hasse principle of f.

In this talk, I will recall Igusa's conjecture for exponential sums and discuss some new progress and open questions relating this conjecture to the singularities of the hypersurface defined by f.

This talk is based on recent joint work with Wim Veys and with Raf Cluckers

K. H. NGUYEN, KU LEUVEN, DEPARTMENT OF MATHEMATICS, CELESTIJNENLAAN 200B, 3001 HEVERLEE, BELGIUM

Email address: kien.nguyenhuu@kuleuven.be