The Hurwitz zeta function is a shifted integer analogue of the Riemann zeta function, for shift parameters $0 < \alpha \leqslant 1$. We consider the integral moments of the Hurwitz zeta function on the critical line $\Re(s) = \frac{1}{2}$. We will focus on rational shift parameters. In this case, the Hurwitz zeta function decomposes as a linear combination of Dirichlet L-functions, which leads us into investigating moments of products of L-functions. Using heuristics from random matrix theory, we conjecture an asymptotic of the same form as the moments of the Riemann zeta function. If time permits, we will discuss the case of irrational shift parameters, which will include some joint work with Winston Heap and Trevor Wooley and some ongoing work with Heap.