In the past century, moments of L-functions have been important in number theory and are well-motivated by a variety of arithmetic applications. In this talk, we will begin with two elementary counting problems of Diophantine nature as motivation, followed by a survey of techniques in the past and the present. The main goal is to demonstrate how period integrals can be used to study moments of automorphic L-functions and uncover the interesting underlying structures, some of which can be modeled by random matrix theory.