On the curved Trilinear Hilbert transform

Victor Lie, Purdue University

Abstract: Building on the (Rank I) LGC-methodology introduced by the speaker and on the novel perspective employed in the time-frequency discretization of the non-resonant bilinear Hilbert–Carleson operator (joint work with C. Benea, F. Bernicot and M. Vitturi), we develop a new, versatile approach—referred to as Rank II LGC—that has as a consequence the resolution of the L^p boundedness of the trilinear Hilbert transform along the moment curve. More precisely, we show that the operator

$$H_C(f_1, f_2, f_3)(x) := \text{p.v.} \int_{\mathbb{R}} f_1(x-t) f_2(x+t^2) f_3(x+t^3) \frac{dt}{t}, \quad x \in \mathbb{R},$$

is bounded from $L^{p_1}(\mathbb{R}) \times L^{p_2}(\mathbb{R}) \times L^{p_3}(\mathbb{R})$ into $L^r(\mathbb{R})$ within the Banach Hölder range $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r}$ with $1 < p_1, p_3 < \infty$, $1 < p_2 \leq \infty$ and $1 \leq r < \infty$.

A crucial difficulty in approaching this problem is the lack of absolute summability for the standard (Rank I) LGC-derived discretized model of the quadrilinear form associated to H_C. In order to overcome this, we design a so-called *correlative* time-frequency model whose control is achieved via the following interdependent elements:

- a sparse-unform decomposition of the input functions adapted to an appropriate time-frequency foliation of the phase-space,
- a structural analysis of suitable maximal "joint Fourier coefficients", and
- a level set analysis with respect to the time-frequency correlation set.

This is joint work with Bingyang Hu.