A remarkable result of Davies shows that an arbitrary measurable set in the plane can be covered by lines in such a way that the union of the lines minus the original set has measure zero. This theorem has an equivalent dual formulation which says that one can find a single set in the plane with given "prescribed" projections in almost every direction, up to measure zero errors. We extend these results to a non-linear setting and prove that a set in the plane can be covered efficiently by translates of a single curve satisfying a mild curvature assumption.