Let $X$ be large and $H$ also large but slightly smaller, and consider $n$ ranging from 1 to $X$. For an arithmetic function $f(n)$ like the $k$-fold divisor function, what is the best mean square approximation of $f(n)$ by a restricted divisor sum (a function of the sort $\sum_{d \mid n, d<H} a_{d}$ )? I hope to explain some of the context around this question and how the answer is connected to random matrix integrals over the unitary group.

