In this talk we will review the Erdős-Kac Theorem on the number of prime factors of a random integer. We will explain how this classical result and its generalizations are proved nowadays. We will then explain how, in recent work with Lasse Grimmelt, we proved an analogous result for a certain distribution on shifted primes, thereby solving a conjecture of Elliott from 2014. Time permitting, we will also review Billingsley's Theorem on the shape of the largest prime factors of a random integer and say what we can say about similar questions in Elliott's set-up.

In more detail, let $\omega(n)$ be the number of prime factors of n. In 1940 Erdős and Kac famously proved that $\omega(n)$ displays gaussian behavior. In 2015, Elliott studied a 'tilted' version of the same problem: let X_n be an integer chosen from 1 to n, with the probability $\mathbb{P}(X_n = k)$ being proportional to $2^{\omega(k)}$. He showed that $\omega(X_n)$ still displays gaussian behavior but with an expectation that is twice as large.

Let Y_n be a random variable, whose distribution is similar to that of X_n, but whose support is restricted to the sparse set $\{n - p : p < n \text{ is a prime}\}$. Elliott conjectured that $\omega(Y_n)$ displays the same gaussian behavior as $\omega(X_n)$ does. We will explain how we solved this conjecture, which turned out to require rather classical tools: the Bombieri-Vinogradov and Brun-Titchmarsh Theorems, which we will recall during the talk.

Towards the end of the talk we will explain what one can say about the largest prime factors of Y_n, which is a question that requires much more recent (and deeper) knowledge about the distribution of primes in arithmetic progressions.

Based on joint work with Lasse Grimmelt.