

Given a polynomial  $F$  in  $\mathbb{R}[x_1, \dots, x_n]$  and some finite set  $A$  of real numbers, we are interested in studying how large is the set

$$F(A, \dots, A) = \{F(a_1, \dots, a_n) : a_1, \dots, a_n \in A\}.$$

A classic result in this setting is the Elekes–Rónyai theorem, which classifies all polynomials  $F$  satisfying

$$|F(A, \dots, A)| \gg |A|^{1+c}$$

for every finite set  $A$  of real numbers, where  $c > 0$  is some absolute constant. In this talk, we are interested in a sum-product type variation of this result. In particular, we want to categorise all polynomials  $F$  such that

$$|F(A, \dots, A)| \gg |A|^n$$

for every finite set  $A$  of real numbers, which produces few products, that is,

$$|A \cdot A| = |a \cdot b : a, b \in A| \ll |A|.$$

This generalises earlier results of Chang, Hanson–Roche–Newton–Zhelezov and Pohoata.