The average behavior of the representation function for sums of two squares, $r(n)$, is well understood. Moreover, its k-th moment—counting the number of solutions to the equation $x_i^2 + y_i^2 = x_j^2 + y_j^2$, for $i, j \leq k$, can also be analyzed using complex-analytic techniques, with the main contribution coming from off-diagonal solutions where $\{x_i, y_i\} \neq \{x_j, y_j\}$ for all $i, j \leq k$.

However, the situation changes significantly when we consider the representation function for sums of two prime squares. Up to the third moment, the main contribution comes from diagonal solutions, a phenomenon known as paucity. Nevertheless, a heuristic argument suggests that for higher moments, this behavior does not persist.

In this talk, we will establish sharp upper bounds on these moments. If time permits, we will also discuss conditional lower bounds on the moments.