SAMPLE PROBLEMS FOR MIDTERM I, MATH265

1-4.Consider the system of linear equations

(a) Write down the augmented matrix associated to this system of linear equations

(b) Apply the Gauss-Jordan elimination to reducing the augmented matrix to the reduced row echelon form.

(c) Describe the set of solutions using (b).

1.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 9 \\ -x_1 + 3x_2 + 2x_3 &= 1 \\ 2x_1 - x_2 + x_3 &= 8 \end{aligned}$$

2.

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 1 \\ 2x_1 + 4x_2 - 2x_3 + 2x_4 &= 2 \\ 5x_1 + 10x_2 - 5x_3 + 5x_4 &= 5 \end{aligned}$$

3.

$$\begin{array}{ccccccc} x_1 & & -x_3 & +3x_4 & = & 0 \\ 2x_1 & +x_2 & -2x_3 & +5x_4 & = & 3 \\ x_1 & -x_2+ & -x_3 & +4x_4 & = & -3 \\ 6x_1 & +3x_2 & -3x_3 & +12x_4 & = & 10 \end{array}$$

4.

$$\begin{array}{cccccc} x_1+ & x_2 + -x_3 & +2x_4 & = & 0 \\ -x_1+ & x_2 + 4x_3 & -3x_4 & = & 0 \\ & x_2 + 5x_3 & +x_4 & = & 0 \end{array}$$

5. Determine whether the following matrices are row equivalent:

$$A = \begin{bmatrix} 2 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & -1 & 0 & 1 \\ 1 & 0 & -2 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

Hint: Find their rref form.

6-8. For which values a (and b) does the system of linear equations [a] have infinitely many solutions; [b] have no solution; [c] have a single solution

6.

$$\begin{array}{rrrcr} x_1 & -x_2 & -x_3 & = & 1 \\ 2x_1 & 2x_2 & +x_3 & = & a \\ 5x_1 & +3x_2+ & x_3 & = & -a^2 \end{array}$$

7.

$$\begin{array}{rrrcr} x_1 & -x_2 & -x_3 & = & 1 \\ & x_2 & +x_3 & = & a \\ x_1 & +2x_2+ & ax_3 & = & a \end{array}$$

8.

$$\begin{array}{rrrcr} x_1 & +2x_2 & +bx_3 & = & a-1 \\ x_1 & +3x_2 & +(a+b)x_3 & = & a \\ 2x_1 & +4x_2- & (a+b)x_3 & = & 2a-1 \end{array}$$

9.Let

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$$

Compute, if possible A^2, AB, B^TA^T, BA

10-11.Determine whether the matrix has an inverse. If yes, find the inverse and write the given matrix as a product of elementary matrices.

10.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

11.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

12. Find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

and solve $Ax = b$, where

$$b = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

13. Answer each of the following as true or false. Briefly justify your answer.

(i) A linear system of four equations in five unknowns has infinitely many solutions.

(ii)A linear system of four equations in three unknowns has no solution.

(iii)If A is nonsingular then $A\mathbf{x} = b$ has a unique solution.

(iv) If A and B are nonsingular and of the same size then AB could be singular.

(v) If $\mathbf{u_1}$ and $\mathbf{u_2}$ are solutions to the homogeneous system $A\mathbf{x} = 0$ the $\mathbf{u_1} + \mathbf{u_2}$ is also a solution to this system.

(vi) If A and B are square matrices and of the same size then $AB = BA$.

(vii) If A and B are square nonsingular matrices and of the same size then $(AB)^{-1} = B^{-1}A^{-1}$

(viii) If A and B are square nonsingular matrices and of the same size then $(AB)^T = B^T A^T$

(ix) If A is symmetric then A^{-1} is also symmetric.

(x)If A^2 is nonsingular then A^3 can be singular.

(xi) If A and B are square nonsingular matrices of the same size then $A + B$ is nonsingular.

14.Let A be a square matrix such that $A^3 = I$. Find A^{-1}

15. For each of the following sets determine if it is a vector subspace

(a) The set of all vectors (x_1, x_2, x_3, x_4) with the property $x_1 - x_2 + x_3 - x_4 = 0$

(b)The set of all vectors (x_1, x_2, x_3, x_4) with the property $x_1 x_2 - x_3 x_4 = 0$

(c)The set of all vectors of the form $v = (a + b, 2a + b, c, a + b + d)$ in R^4.

(d)The set of all vectors of the form $v = (a + b, 2a + b, c + 1, a + b + d)$ in R^4.

16. Explain why the following sets are not vector spaces.

a) $W = \{(a_1, a_2) : a_1, a_2 \in R\}$
$(a_1, a_2) \oplus (b_1, b_2) = (2a_1 + 2b_1, 2a_2 + 2b_2)$,
$c \odot (a_1, a_2) = (ca_1, ca_2)$

b) $W = \{(a_1, a_2) : a_1, a_2 \in R\}$
$(a_1, a_2) \oplus (b_1, b_2) = (a_1 + b_1, 2a_2 + b_2)$,
$c \odot (a_1, a_2) = (ca_1, ca_2)$

c)b) $W = \{(a_1, a_2) : a_1, a_2 \in R\}$
$(a_1, a_2) \oplus (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$
$c \odot (a_1, a_2) = (ca_1, 2ca_2)$

17. Express, if possible, one of the vectors of the following set as a linear combination of the others: $v_1 = (1, -1, 0)$, $v_2 = (0, 2, 1)$, $v_3(1, -2, 3)$, $v_4 = (3, 6, 2)$.

18.For what values of d is the vector $(2, 1, d)$ in the span of $(1, 2, 1)$ and $(2, 5, 1)$.

19. Let $S = \{v_1, v_2, v_3\}$, where $v_1 := (1, 2, 2)$, $v_2 := (3, 2, 1)$ $v_3 = (7, 6, 4)$. Determine if S is lineraly independent. Find a basis for the subspace $W = spanS$ of R^3 and its dimension . Determine whether $u = (1, 6, 5)$ is in W.

20.For the problem (a) (b) (c) determine whether the given set of vectors is linearly inedpendent or linearly dependent:

(a) $\{(1,0,-1),(2,1,1)\}$
(b) $\{(1,0,-1),(2,1,-1),(0,1,3)\}$
(c) $\{(1,0,-1),(1,3,4),(2,1,-1),(0,1,3)\}$

For the problem (d) (e) (f) determine whether the given set of vectors spans R^3
(d) $\{(1,1,-1),(2,1,1)\}$
(e) $\{(1,0,0),(1,3,4),(2,1,0),(0,1,3)\}$
(f) $\{(1,0,0),(1,3,4),(2,1,0)\}$

For the problem (g) (h) (i) determine whether the given set of vectors form a basis of R^3
(g)$(1,2,0)$, $(1,1,1)$, $(1,0,1)$.
(h)$(1,2,0)$, $(1,1,1)$, $(2,3,1)$
(i)$(1,2,0)$, $(1,1,1)$

21. Let $W = \{(1,-1,-1,2),(0,1,2,0),(2,-1,0,2),(1,0,1,2)\}$ be a set of vectors in R^4. Find a basis of subspace of $spanW$ consisting of some vectors in W.

22.Let W be the subspace of R^4 consisting of all vectors of the form $(b+c, a-b, 2b-a+c, a+c)$. Find a basis of W and its dimension.

23.Find a basis of of $\{(x,y,z) \mid x+y+2z=0\}$