PROJECTS AND PAPERS

A Fibonacci Numbers, the Golden Ratio, and Phyllotaxis

In this chapter we mentioned that Fibonacci numbers and the golden ratio often show up in both the plant and animal worlds. In this project you are asked to expand on this topic.

1. Give several detailed examples of the appearance of Fibonacci numbers and the golden ratio in the plant world. Include examples of branch formation in plants, leaf arrangements around stems, and seed arrangements on circular seedheads (such as sunflower heads).
2. Discuss the concept of *phyllotaxis*. What is it, and what are some of the mathematical theories behind it?

(*Notes*: The literature on Fibonacci numbers, the golden ratio, and phyllotaxis is extensive. A search on the Web should provide plenty of information. Two excellent Web sites on this subject are Ron Knott's site at the University of Surrey, England (http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci), and the site *Phyllotaxis: An Interactive Site for the Mathematical Study of Plant Pattern Formation* (http://www.math.smith.edu/~phyllo/) based at Smith College.

B The Golden Ratio in Art, Architecture, and Music

It is often claimed that from the time of the ancient Greeks through the Renaissance to modern times, artists, architects, and musicians have been fascinated by the golden ratio. Choose one of the three fields (art, architecture, or music) and write a paper discussing the history of the golden ratio in that field. Describe famous works of art, architecture, or music in which the golden ratio is alleged to have been used. How? Who were the artists, architects, and composers?

Be forewarned that there are plenty of conjectures, unsubstantiated historical facts, controversies, claims, and counterclaims surrounding some of the alleged uses of the golden ratio and Fibonacci numbers. Whenever appropriate, you should present both sides to a story.

C Logarithmic Spirals

In this project you are to investigate logarithmic spirals. Y should explain what a logarithmic spiral is and how it is re ed to Fibonacci numbers and the golden ratio, and y should reference several examples of where they occur in natural world.

D Figurate Numbers

The *triangular* numbers are numbers in the sequence 1, 3, 6, 15, ... (The Nth **triangular** number T_N is given by the s $1 + 2 + 3 + \cdots + N$.) In a similar way, *square*, *pentago* and *hexagonal* numbers can be defined. In this project you to investigate these types of numbers (called *figurate* numbe give some of their more interesting properties, and discuss relationship between these numbers and *gnomons*.

E The Golden Ratio Hypothesis

A long-held belief among those who study how humans ceive the outside world (mostly psychologists and psych ologists) is that the *golden ratio* plays a special and promi role in the human interpretation of "beauty." Shapes objects whose proportions are close to the golden ratio believed to be more pleasing to human sensibilities t those that are not. This theory, generally known as *the go ratio hypothesis*, originated with the experiments of fan psychologist Gustav Fechner in the late 1800s. In a cl experiment, Fechner showed rectangles of various pro tions to hundreds of subjects and asked them to choose results showed that the rectangles that were close to proportions of the golden ratio were overwhelmingly ferred over the rest. Since Fechner's original experin there has been a lot of controversy about the golden hypothesis, and many modern experiments have cast ous doubts about its validity.

Write a paper describing the history of the golden hypothesis. Start with a description of Fechner's origin periment. Follow up with other experiments duplic Fechner's results and some of the more recent experin that seem to disprove the golden ratio hypothesis. Con with your own analysis.

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95. According to a *Philadelphia Inquirer* finance column by Jeff Brown on November 1, 2005, "Borrow \$100,000 with a 6% fixed-rate mortgage and you'll pay nearly \$116,000 in interest over 30 years. Put an extra \$100 a month into principal payments and you'd pay just \$76,000—and be done with mortgage payments nine years earlier."

(a) Verify that the increase in the monthly payment that is needed to pay off the mortgage in 21 years is indeed close to \$100 and that roughly \$40,000 will be saved in interest.

(b) How much should the monthly payment be increased so as to pay off the mortgage in 15 years? How much interest is saved in doing so?

(c) How much should the monthly payment be increased so as to pay off the mortgage in t years ($t < 30$)? Express the answer in terms of t.

96. Sam started his new job as the mathematical consultant for the XYZ Corporation on July 1, 2005. The company retirement plan works as follows: On July 1, 2006, the company deposits \$1000 in Sam's retirement account, and each year thereafter on July 1 the company deposits the amount deposited the previous year plus an additional 6%. The last deposit is to be made on July 1, 2035. In addition, the retirement account earns an annual interest rate of 6% compounded monthly. On July 1, 2035, all deposits and interest paid to the account will stop. What is the future value of Sam's retirement account?

97. Perpetuities. A perpetuity is a constant stream of identical annual payments with no end. The present value of a perpetuity of \$$C$, given an annual interest rate p (expressed as a decimal), is given by the formula

$$P = C \cdot \frac{1}{(1+p)} + C \cdot \frac{1}{(1+p)^2} + C \cdot \frac{1}{(1+p)^3} + \cdots$$

(a) Use the geometric sum formula to simplify the sum

$$C \cdot \frac{1}{(1+p)} + C \cdot \frac{1}{(1+p)^2} + C \cdot \frac{1}{(1+p)^3} + \cdots + C \cdot \frac{1}{(1+p)^T}$$

(b) Explain why as T gets larger, the value obtained in (a) gets closer and closer to C/p. (Assume that $0 < p < 1$.)

PROJECTS AND PAPERS

A The Many Faces of *e*

The irrational number e has many remarkable mathematical properties. In this project you are asked to present and discuss five of the more interesting mathematical properties of e. For each property, give a historical background (if possible), a simple mathematical explanation (avoid technical details if you can), and a real-life application (see, for example, reference 3).

B Growing Annuities

Over time, the fixed payment from an annuity (such as a retirement account) will get you fewer and fewer goods and services. If prices rise 3% a year, items that cost \$1000 today will cost more than \$1300 in 10 years and more than \$1800 in 20 years. To combat this phenomenon, *growing* annuities—in which the payments rise by a fixed percentage, say 3%, each year—have been developed. In this project you are to discuss the mathematics behind growing annuities. (See references 2 and 5.)

C A Future Home Buyer's Must-Do Project

In this project, you will investigate the process of buying a house. For the purposes of this project, we will assume that you are going to graduate at the end of this semester, get a job, and begin saving for your first house. You will plan to buy your house five years after graduation. At that time, you will use the money you've saved as a down payment and take out a loan for the rest of the cost of the house.

As part of the project, you should address each of the following:

1. *Your projected salary for the first five years after you graduate.* Describe what kind of a job you expect to have and what starting monthly salary is realistic for that type of job. Project the salary over five years, including raises and cost-of-living increases. (Support all your salary assumptions with data and cite your sources.)
2. *Your savings and investments over the five-year period.* For the purposes of this project, assume that 10% of the monthly salary you projected in Step 1 will be invested in a deferred annuity. Research the types of deferred annuities available to you and describe the terms, fees, and APR. Use the amount of money you will be able to save each month, the interest rate you have found, and your knowledge of annuities to compute how much money you will have after five years. Don't forget to deduct any state and federal taxes you will be paying on your income from the annuities. Estimate your taxes on

income from the annuities at 20% federal and 7% state. (If you expect to have a job in a no-income-tax state, disregard the 7% state tax.)

3. *How much you can afford to spend on a home and what your monthly mortgage expenses will be.* To determine how much you can afford to spend on a home, use your estimate of how much you have saved for a down payment and assume that your down payment is 15% of the cost of the home. Research the types of home loans currently available to someone with your credit rating and income (assume the income projected at the end of the five-year period). Describe the terms of a loan you might be able to take out, including the interest rate, type and length of loan, and loan origination fees. Once you found a loan, project your monthly mortgage payments (be sure to include property taxes and insurance). Once again, cite all the sources for your data and show all the formulas you are using for your calculations.

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Mortgage Calculators on the Web

- http://www.amortization-calc.com/
- http://calculator-loan.info/
- http://www.mortgageloan.com/calculator/amortization-calculator
- http://www.vertex42.com/ExcelTemplates/excel-amortization-spreadsheet.html
- https://www.federalreserve.gov/apps/mortcalc/

Notes: (1) Your best bet is to look at wallpaper patterns and borders at a wallpaper store or gift-wrapping paper and ribbons at a paper store. You will have to do some digging—a few of the wallpaper-pattern symmetry types are hard to find. (2) For ideas, you may want to visit Steve Edwards's excellent Web site *Tiling Plane and Fancy* at *http://www2.spsu.edu/math/tile/index.htm.*

B Three-Dimensional Rigid Motions

For two-dimensional objects, we have seen that every rigid motion is of one of four basic types. For three-dimensional objects moving in three-dimensional space, there are *six* possible types of rigid motion. Specifically, every rigid motion in three-dimensional space is equivalent to a *reflection*, a *rotation*, a *translation*, a *glide reflection*, a *rotary reflection*, or a *screw displacement*.

Prepare a presentation on the six possible types of rigid motions in three-dimensional space. For each one give a precise definition of the rigid motion, describe its most important properties, and give illustrations as well as real-world examples.

C Penrose Tilings

In the mid-1970s, British mathematician Roger Penrose made a truly remarkable discovery—it is possible to cover the plane using *aperiodic tilings*, something like a wallpaper pattern with a constantly changing motif. (See the biographical profile on page 421.) One of the simplest and most surprising aperiodic tilings discovered by Penrose is based on just two shapes—the two rhombi shown in Figure 11.91.

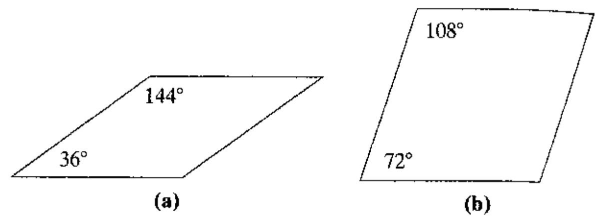

FIGURE 11-91

Prepare a presentation discussing and describing Penrose's tilings based on figures (a) and (b). Include in your presentation the connection between figures (a) and (b) and the golden ratio as well as the connection between Penrose tilings and quasicrystals.

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69. (a) Show that the complex number $s = -0.25 + 0.25i$ is in the Mandelbrot set.

(b) Show that the complex number $s = -0.25 - 0.25i$ is in the Mandelbrot set. [*Hint*: Your work for (a) can help you here.]

70. Show that the Mandelbrot set has a reflection symmetry. (*Hint*: Compare the Mandelbrot sequences with seeds $a + bi$ and $a - bi$.)

Exercises 71 through 73 refer to the concept of ***fractal dimension****. The fractal dimension of a geometric fractal consisting of N self-similar copies of itself each reduced by a scaling factor of S is $D = \log N/\log S$. (The fractal dimension is described in a little more detail in Project A below.)*

71. Compute the fractal dimension of the *Koch curve*.

72. Compute the fractal dimension of the *Sierpinski carpet*. (The Sierpinski carpet is discussed in Exercises 23 through 26.)

73. Compute the fractal dimension of the *Menger sponge*. (The Menger sponge is discussed in Exercises 57 and 58.)

PROJECTS AND PAPERS

A Fractal Dimension

The dimensions of a line segment, a square, and a cube are, as we all learned in school, 1, 2, and 3, respectively. But what is the dimension of the Sierpinski gasket?

The line segment of size 4 shown in Fig. 12-44(a) is made of four smaller copies of itself each scaled down by a factor of four; the square shown in Fig. 12-44(b) is made of $16 = 4^2$ smaller copies of itself each scaled down by a factor of four; and the cube shown in Fig. 12-44(c) is made of $64 = 4^3$ smaller copies of itself each scaled down by a factor of four. In all these cases, if N is the number of smaller copies of the object reduced by a scaling factor S, the dimension D is the exponent to which we need to raise S to get N (i.e., $N = S^D$). If we apply the same argument to the Sierpinski gasket shown in Fig. 12-44(d), we see that the Sierpinski gasket is made of $N = 3$ smaller copies of itself and that each copy has been reduced by a scaling factor $S = 2$. If we want to be consistent, the dimension of the Sierpinski gasket should be the exponent D in the equation $3 = 2^D$. To solve for D you have to use *logarithms*. When you do, you get $D = \log 3/\log 2$. Crazy but true: The dimension of the Sierpinski gasket is not a whole number, not even a rational number. It is the irrational number $\log 3/\log 2$ (about 1.585)!

For a geometric fractal with exact self-similarity, we will define its dimension as $D = \log N/\log S$, where N is the number of self-similar pieces that the parent fractal is built out of and S is the scale by which the pieces are reduced (if the pieces are one-half the size of the parent fractal $S = 2$, if the pieces are one-third the size of the parent fractal $S = 3$, and so on).

In this project you should discuss the meaning and importance of the concept of dimension as it applies to geometric fractals having exact self-similarity.

B Fractals and Music

The hallmark of a fractal shape is the property of *self-similarity*—there are themes that repeat themselves (either exactly or approximately) at many different scales. This type of repetition also works in music, and the application of fractal concepts to musical composition has produced many intriguing results.

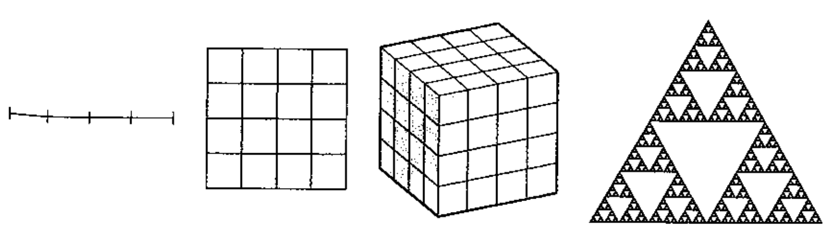
Write a paper discussing the connections between fractals and music.

C Book Review: *The Fractal Murders*

If you enjoy mystery novels, this project is for you.

The Fractal Murders by Mark Cohen (Muddy Gap Press, 2002) is a *whodunit* with a mathematical backdrop. In addition to the standard elements of a classic murder mystery (including a brilliant but eccentric detective), this novel has a fractal twist: The victims are all mathematicians doing research in the field of fractal geometry.

Read the novel and write a review of it. Include in your review a critique of both the literary and the mathematical merits of the book. To get some ideas as to how to write a good book review, you should check out the *New York Times*



(a) (b) (c) (d)

FIGURE 12-44

Book Review section, which appears every Sunday in the *New York Times* (*www.nytimes.com*).

D Fractal Antennas

One of the truly innovative practical uses of fractals is in the design of small but powerful antennas that go inside wireless communication devices such as cell phones, wireless modems, and GPS receivers. The application of fractal geometry to antenna design follows from the discovery in 1999 by radio astronomers Nathan Cohen and Robert Hohlfeld of Boston University that an antenna that has a self-similar shape has the ability to work equally well at many different frequencies of the radio spectrum.

Write a paper discussing the application of the concepts of fractal geometry to the design of antennas.

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(d) Explain why, if $0 < p_0 < 1$ and $0 < r < 4$, then $0 < p_N < 1$, for every positive integer N.

32. Suppose that $r > 3$. Using the logistic growth model, find a population p_0 such that $p_0 = p_2 = p_4 \ldots$, but $p_0 \neq p_1$.

33. Show that if $P_0, P_1, P_2, \ldots$ is an arithmetic sequence, then $2^{P_0}, 2^{P_1}, 2^{P_2}, \ldots$ must be a geometric sequence.

PROJECTS AND PAPERS

A The Malthusian Doctrine

In 1798, Thomas Malthus wrote his famous *Essay on the Principle of Population*. In this essay, Malthus put forth the principle that population grows according to an exponential growth model, whereas food and resources grow according to a linear growth model. Based on this doctrine, Malthus predicted that humankind was doomed to a future where the supply of food and other resources would be unable to keep pace with the needs of the world's population.

Write an analysis paper detailing some of the consequences of Malthus's doctrine. Does the doctrine apply in a modern technological world? Can the doctrine be the explanation for the famines in sub-Saharan Africa? Discuss the many possible criticisms that can be leveled against Malthus's doctrine. To what extent do you agree with Malthus's doctrine?

B The Logistic Equation and the United States Population

The logistic growth model, first discovered by Verhulst, was rediscovered in 1920 by the American population ecologists Raymond Pearl and Lowell Reed. Pearl and Reed compared the population data for the United States between 1790 and 1920 with what would be predicted using a logistic equation and found that the numbers produced by the equation and the real data matched quite well.

In this project, you are to discuss and analyze Pearl and Reed's 1920s paper (reference 10). Here are some suggested questions you may want to discuss: Is the logistic model a good model to use with human populations? What might be a reasonable estimate for the carrying capacity of the United States? What happens with Pearl and Reed's model when you expand the census population data all the way to the latest population figures available? Note: Current and historical U.S. population data can be found at *www.census.gov*.

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21. Given a graph G with n vertices show that there is a way to order the vertices in the order $v_1, v_2, \ldots, v_n$ so that when the greedy algorithm is applied to the vertices in that order, the resulting coloring of the graph uses $\chi(G)$ colors. (*Hint: Assume a coloring of the graph with* $\chi(G)$ *colors and work your way backward to find an ordering of the vertices that produces that particular coloring.*)

22. Let S denote the Sudoku graph discussed in this mini-excursion.

(a) Explain why every vertex of S has degree 20.

(b) Explain why S has 810 edges. (*Hint: See Euler's theorems in Chapter 5.*)

23. If you haven't done so yet, try the Sudoku puzzle given in Fig. ME2-10.

PROJECTS

Scheduling Committee Meetings of the U.S. Senate

This project is a surprising application of the graph coloring techniques we developed in this mini-excursion and involves an important real-life problem: scheduling meetings for the standing committees of the United States Senate.

The U.S. Senate has 16 different standing committees that meet on a regular basis. The business of these committees represents a very important part of the Senate's work, since legislation typically originates in a standing committee and only if it gets approved there moves on to the full Senate. Scheduling meetings of the 16 standing committees is complicated because many of these committees have members in common, and in such cases the committee meetings cannot be scheduled at the same time. An easy solution would be to schedule the meetings of the committees all at different time slots that do not overlap, but this would eat up the lion's share of the Senate's schedule, leaving little time for the many other activities that the Senate has to take on. The optimal solution would be to schedule committees that do not have a member in common for the same time slot and committees that do have a member in common for different time slots. This is where graph coloring comes in.

Imagine a graph where the vertices of the graph represent the 16 committees, and two vertices are adjacent if the corresponding committees have one or more members in common. (For convenience, call this graph the *Senate Committees graph.*) If we think of the possible time slots as colors, a k-coloring of the Senate Committees graph gives a way to schedule the committee meetings using k different time slots. In this project you are to find a meetings schedule for the 16 standing committees of the U.S. Senate using the fewest possible number of time slots.

[*Hints*: (1) Find the membership lists for each of the 16 standing committees of the U.S. Senate. For the most up-to-date information, go to *http://www.senate.gov* and click on the Committees tab. (2) Create the Senate Committees graph. (You can use a spreadsheet instead of drawing the graph—it is quite a large graph—and get all your work done through the spreadsheet.) (3) List the vertices of the graph in decreasing order of degrees and use the greedy algorithm to color the graph.]

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KEY CONCEPTS

arithmetic mean, **153**
Huntington-Hill method, **152**
mean, **153**
method of equal proportions, **152**

PROJECTS AND PAPERS

A Dean's Method

Dean's method is a method almost identical to the Huntington-Hill method (and thus to Webster's method) that uses the *harmonic means* of the lower and upper quotas (see Exercise 22) as the cutoff points for rounding. In this project you are to discuss Dean's method by comparing it to the Huntington-Hill method (in a manner similar to how the Huntington-Hill method was discussed in the mini-excursion by comparison to Webster's method). Give examples that illustrate the difference between Dean's method and the Huntington-Hill method. You should also include a brief history of Dean's method.

B *Montana v. U.S. Department of Commerce* (U.S. District Court, 1991); *U.S. Department of Commerce v. Montana* (U.S. Supreme Court, 1992)

Write a paper discussing these two important legal cases concerning the apportionment of the U.S. House of Representatives. In this paper you should (1) present the background preceding Montana's challenge to the constitutionality of the Huntington-Hill method, (2) summarize the arguments presented by Montana and the government in both cases, (3) summarize the arguments given by the District Court in ruling for Montana, and (4) summarize the arguments given by the Supreme Court in unanimously overturning the District Court ruling.

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