

Lab #10: SIR Model of a Disease

Expectations

1.
 - a) Use pplane to plot the system (*) for $0 < t < 4$. Describe what happens to S and I as $t \rightarrow 4$.
 - b) Approximate the maximum number of infected people in the first 4 months. Estimate when this occurs.
 - c) Is the number of susceptibles ever the same as the number of infected during this interval? If so, when does this occur?
 - d) Estimate $S(2)$, $I(2)$, and $I(3)$.
2. Suppose at $t=2$, the virus mutates so that $a=0.5$. Estimate I one month after the virus mutates. Compare this with $I(3)$ from 1.d). Explain the difference.
3. Estimate $S(t^*)$ where t^* is the time at which the number of infected is at its maximum with the given initial conditions. Fill in the table.
Explain why you should expect that the system (*) has a maximum I value at $S(t^*)=b/a$. Does the table agree with this?
4. What is $s + i + r$?
 - a) Using (*), derive the corresponding system for s , i , and r .
 - b) Use pplane to determine the validity of the source's 3 statements.
5.
 - a) Using (*), show that $dS/dI = aSI/(bI - aSI)$.
 - b) With the given initial conditions, show that:
$$I(t) = I_0 + (S_0 - S(t)) + (b/a)\ln(S(t)/S_0).$$

Plots: 1.a), 1.c), 2., 3 plots for 4.b)