ode45 - Differential Equation Solver

This routine uses a variable step Runge-Kutta Method to solve differential equations numerically. The syntax for **ode45** for first order differential equations and that for second order differential equations are basically the same. However, the .m files are quite different.

I. First Order Equations $\begin{cases} y' = f(t,y) \\ y(t_0) = y_0 \end{cases}$

A. Create a .m file for $f(t,y)$ (see the tutorial on numerical methods and m files on how to do this). Save file as, for example, **yp.m** .

B. Basic syntax for **ode45** . At a MATLAB prompt type :

```
[t,y]=ode45('yp',[t0,tf],y0);
```

(your version of **ode45** may not require backets around t0, tf)

$$\begin{cases} \texttt{yp} = \text{ the .m file of the function } f(t,y) \text{ saved as yp.m} \\ \texttt{t0, tf} = \text{ initial and terminal values of } t \\ \texttt{y0} = \text{ initial value of } y \text{ at } t_0 \end{cases}$$

C. For example, to numerically solve $\begin{cases} t^2 y' = y + 3t \\ y(1) = -2 \end{cases}$ over $1 \le t \le 4$:

- Create and save the file yp.m for the function $\frac{1}{t^2}(y+3t)$.
- At a MATLAB prompt type:

```
[t,y]=ode45('yp',[1,4],-2);
```

(your version of **ode45** may not require backets around [1,4])

- To print results type : `[t,y]`
- To plot results type : `plot(t,y)`
- To plot results type with a '+' symbol : `plot(t,y,'+')`

II. Second Order Equations $\begin{cases} y'' + p(t)\,y' + q(t)\,y = g(t) \\ y(t_0) = y_0 \\ y'(t_0) = y_1 \end{cases}$

A. First convert 2^{nd} order equation to an equivalent system of 1^{st} order equations. Let $x_1 = y,\ x_2 = y'$:

$$\begin{cases} x_1' = x_2 \\ x_2' = -q(t)\,x_1 - p(t)\,x_2 + g(t) \end{cases}, \text{ where } x_1(t_0) = y_0,\ x_2(t_0) = y_1 .$$

B. Create and save a .m file which will return a <u>vector-valued</u> function. This is a little tricky so here is a specific example. Suppose the system is as below and $0 \le t \le 4$

$$\begin{cases} x_1' = x_2 \\ x_2' = -t\,x_1 + e^t\,x_2 + 3\sin 2t \end{cases}, \text{ where } x_1(0) = 2,\ x_2(0) = 8 .$$

- Create the following function file and save it as **F.m** :

```
function xp=F(t,x)
xp=zeros(2,1);   % since output must be a column vector
xp(1)=x(2);
xp(2)=-t*x(1)+exp(t)*x(2)+3*sin(2*t);
```

(Don't forget the ";" after each line.)

- Basic syntax for **ode45** . At MATLAB prompt, type :

```
[t,x]=ode45('F',[t0,tf],[x10,x20]);
```

$$\begin{cases} \texttt{F= the .m file of the vector-function saved as above} \\ \texttt{t0, tf = initial and terminal values of } t \\ \texttt{x10 = initial value of } x_1 \texttt{ at } t_0 \texttt{ : x10} = x_1(t_0) \\ \texttt{x20 = initial value of } x_2 \texttt{ at } t_0 \texttt{ : x20} = x_2(t_0) \end{cases}$$

(The example above becomes : `[t,x]=ode45('F',[0,4],[2,8]);`

- Since $x_1(t) = y$, to print out the values of the solution y for $t_0 \leq t \leq t_f$, at a MATLAB prompt type: `[t,x(:,1)]`

  To plot the solution on a graph t vs y, type: `plot(t,x(:,1))`

  (since the vector $\mathbf{x}$ has 1^{st} component $x_1 = y$ and 2^{nd} component $x_2 = y'$.)

- To plot x_1 vs x_2 (phase plane) type: `plot(x(:,1),x(:,2))`