MA 523/Fall 2015
Homework 1
(Due Tuesday, September 8 in class or 3pm in MATH 714)

1. Let $U \subset \mathbb{R}^n$ be a bounded domain with smooth boundary $\partial U$. Let $\nu$ denote the outward unit normal of $\partial U$. For two smooth functions $f, g$ on $U$. Prove the second Green’s identity:

$$\int_{U} (f \Delta g - g \Delta f) = \int_{\partial U} (f \frac{\partial g}{\partial \nu} - g \frac{\partial f}{\partial \nu}).$$

2. Write down an explicit formula for a solution $u$ to:

$$\begin{cases} u_t + b \cdot Du + cu = 0 & \text{in } \mathbb{R}^n \times (0, +\infty) \\ u = g & \text{on } \mathbb{R}^n \times \{t = 0\}. \end{cases}$$

Here $c \in \mathbb{R}$ and $b \in \mathbb{R}^n$ are constants.

3. Solve the following equation:

$$au_x + bu_y + cu = 0 \text{ in } \mathbb{R}^2; \quad u(x, 0) = f(x) \text{ on } \mathbb{R},$$

where $a, b, c \in \mathbb{R}$ are non-zero constants, and $f$ is a given function.

4. Solve the following equation:

$$u_x - u_y + 5u = e^{3x-4y} \text{ in } \mathbb{R}^2; \quad u(x, 0) = 0 \text{ on } \mathbb{R}.$$