# QUIZ 10: LESSON 13
# FEBRUARY 14, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

Consider the region bounded by $y = \dfrac{3}{x}$ and $y = -x + 4$.

$\frac{3}{x} = -x + 4$

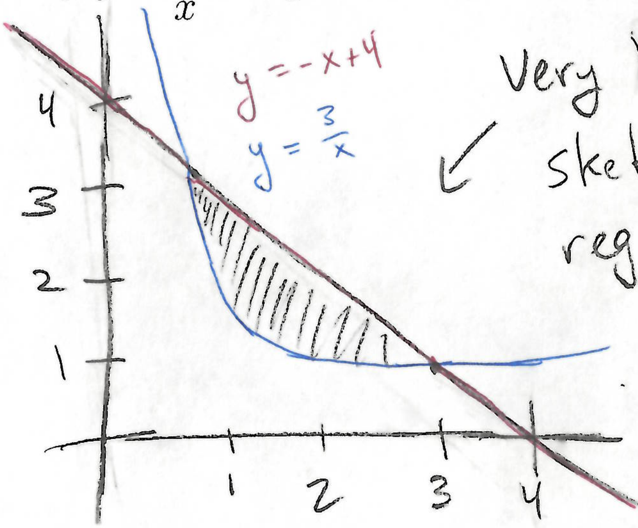
$3 = -x^2 + 4x$

$0 = -x^2 + 4x - 3$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$\Rightarrow x = 1, 3$




Very rough sketch of region

**1.** [5 pts] Find the volume of the solid obtained by revolving the region about the $x$-axis.

Our bounds will be $x = 1, 3$. The outer radius is $-x+4$ the inner radius is $\frac{3}{x}$. So

Volume: $\displaystyle\int_1^3 \pi\left[\underbrace{(-x+4)^2}_{\text{outer}} - \underbrace{\left(\tfrac{3}{x}\right)^2}_{\text{Inner}}\right]dx = \pi\int_1^3\left[x^2 - 8x + 16 - \frac{9}{x^2}\right]dx$

$\displaystyle = \pi\int_1^3 \left[x^2 - 8x + 16 - 9x^{-2}\right]dx = \pi\left[\frac{1}{3}x^3 - 4x^2 + 16x + 9x^{-1}\right]_1^3$

$= \pi\left[\frac{1}{3}(3)^3 - 4(3)^2 + 16(3) + \frac{9}{3} - \left(\frac{1}{3}(1)^3 - 4(1)^2 + 16(1) + \frac{9}{1}\right)\right]$

$= \pi\left[9 - 36 + 48 + 3 - \frac{1}{3} + 4 - 16 - 9\right] = \pi\left[3 - \frac{1}{3}\right] =$

$= \boxed{\pi\left[\frac{8}{3}\right]}$

**2.** [5 pts] Find the volume of the solid obtained by revolving the region about the $y$-axis.

Because we are revolving about the $y$-axis, we need to convert our radii into functions of $y$ and change our bounds in terms of $y$:

$$y = \frac{3}{x} \Rightarrow x = \frac{3}{y}, \quad y = -x+4 \Rightarrow x = -y+4$$

$$\frac{3}{y} = -y+4 \Rightarrow y = 1, 3$$

Volume: $$\int_1^3 \pi\Big[\underbrace{(-y+4)^2}_{\text{Outer}} - \underbrace{\left(\tfrac{3}{y}\right)^2}_{\text{Inner}}\Big]dy$$

Observe that, up to replacing $y$ by $x$, this is exactly the integral we computed before. So Vol = $\boxed{\frac{8}{3}\pi}$

<u>Note</u>: This is not true in general, which is to say revolving the same region about the $x$- and $y$-axis typically results in different volumes. But, in this instance, we were saved some computations.