QUIZ 11 SOLUTIONS: LESSONS 14-15
FEBRUARY 21, 2018

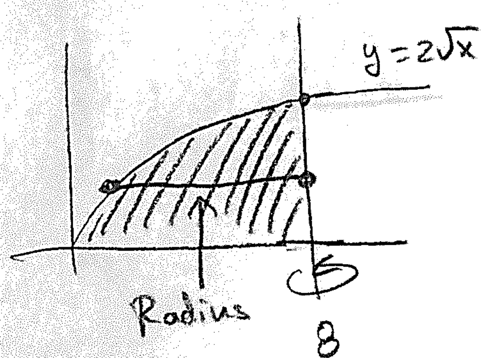
Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Set up but **do not evaluate** the integral that represents the volume of the solid obtained by revolving he region enclosed by

$$y = 2\sqrt{x},\ y = 0, \text{ and } x = 8$$

about the line $x = 8$.

This is a disk method about a vertical line. So



$y = 2\sqrt{x} \Rightarrow \frac{1}{2}y = \sqrt{x}$

$\Rightarrow \frac{1}{4}y^2 = x$

Bounds: $8 = \frac{1}{4}y^2$ Radius: $8 - \frac{1}{4}y^2$

$\Rightarrow 32 = y^2$

$\Rightarrow \sqrt{32} = y$

Vol: $\boxed{\int_0^{\sqrt{32}} \pi \left(8 - \frac{1}{4}y^2\right)^2 dy}$

2. [5 pts] Determine if

$$\int_1^2 \frac{4}{\sqrt{x-1}}$$

converges or diverges. If it converges, find its value.

$\frac{4}{\sqrt{x-1}}$ does not exist at $x=1$. So this is what we make into our limit.

$$\int_1^2 \frac{4}{\sqrt{x-1}}\,dx = \lim_{s \to 1^+} \int_s^2 \frac{4}{\sqrt{x-1}}\,dx$$

Note: We want $s > 1$ else $[s,2]$ still includes 1

Evaluating the integral:

$$\int_s^2 \frac{4}{\sqrt{x-1}}\,dx = \int_s^2 4(x-1)^{-1/2}\,dx = 4\left(\frac{1}{-\frac{1}{2}+1}\right)(x-1)^{-\frac{1}{2}+1}\Big|_s^2$$

$$= 4\left(\frac{1}{\frac{1}{2}}\right)(x-1)^{1/2}\Big|_s^2$$

$$= 8(x-1)^{1/2}\Big|_s^2$$

$$= 8(2-1)^{1/2} - 8(s-1)^{1/2}$$

$$= 8(1) - 8(s-1)^{1/2}$$

So, $\lim_{s \to 1^+} \int_s^2 \frac{4}{\sqrt{x-1}}\,dx = \lim_{s \to 1^+} \left[8 - 8(s-1)^{1/2}\right] = 8$

Therefore, the integral $\boxed{\text{converges}}$ and

$$\int_1^2 \frac{4}{\sqrt{x-1}}\,dx = \boxed{8}.$$