QUIZ 12 SOLUTIONS: LESSONS 16-17
FEBRUARY 26, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Does $\sum_{n=1}^{\infty}\left(-\frac{2}{3}\right)^n$ converge? If so, compute its value. Give a reason for your answer.

Need to change to n

$$\sum_{n=1}^{\infty}\left(-\frac{2}{3}\right)^n = \sum_{n+1=1}^{\infty}\left(-\frac{2}{3}\right)^{n+1} = \sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n\left(-\frac{2}{3}\right)^1$$

change to 0, replace n by n+1

$$= \sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)^n$$

c r

Because $|r| = \left|-\frac{2}{3}\right| < 1$, this series converges. By the geo. series formula,

$$\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)^n = \frac{-\frac{2}{3}}{1-\left(-\frac{2}{3}\right)} = \frac{-\frac{2}{3}}{1+\frac{2}{3}} = \frac{-\frac{2}{3}}{\frac{5}{3}} =$$

c r r

$$= -\frac{2}{3} \div \frac{5}{3}$$

$$= -\frac{2}{3} \times \frac{3}{5}$$

$$= \boxed{-\frac{2}{5}}$$

2. [5 pts] Assuming the following pattern continues indefinitely, determine whether the following converges and, if it does, find its sum:

$$\frac{121}{4} - \frac{11}{2} + 1 - \frac{2}{11} + \frac{4}{121} - \cdots$$

Round your answer to the nearest hundredth.

$r = \frac{-2}{11}$ because

$$\frac{121}{4} + \left(\frac{-11}{2}\right) + 1 + \left(\frac{-2}{11}\right) + \frac{4}{121} - \cdots$$

$$\left(\frac{-2}{11}\right)^{-2} \quad \left(\frac{-2}{11}\right)^{-1} \quad \left(\frac{-2}{11}\right)^{0} \quad \left(\frac{-2}{11}\right)^{1} \quad \left(\frac{-2}{11}\right)^{2}$$

So, this series is given by $\sum_{n=-2}^{\infty} \left(\frac{-2}{11}\right)^n$

↑ Need to change to 0

Replace n by n-2 to get ← Need to change to n

$$\sum_{n=-2}^{\infty} \left(\frac{-2}{11}\right)^n = \sum_{n=0}^{\infty} \left(\frac{-2}{11}\right)^{n-2} = \sum_{n=0}^{\infty} \left(\frac{-2}{11}\right)^n \left(\frac{-2}{11}\right)^{-2}$$

Since $|r| = \left|\frac{-2}{11}\right| < 1$, this series converges. By the geo. series Formula, ← c

$$\sum_{n=0}^{\infty} \left(\frac{-2}{11}\right)^{-2} \left(\frac{-2}{11}\right)^n = \frac{\left(\frac{-2}{11}\right)^{-2}}{1 - \left(\frac{-2}{11}\right)} = \frac{\frac{121}{4}}{1 + \frac{2}{11}} = \frac{\frac{121}{4}}{\frac{13}{11}}$$

c r r

$$= \frac{121}{4} \div \frac{13}{11}$$

$$= \frac{121}{4} \times \frac{11}{13}$$

$$\approx \boxed{25.60}$$